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AUTHOR	Rexroat, Melvin E.
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ABSTRACT

This module for preservice elementary school teachers contains an extensive collection of ideas and materials for planning, modifying, and implementing mathematics laboratory activities. Objectives for the module are stated, pre- and post-assessment activities are listed, and six instructional activities to be completed by the preservice teachers are specified. The module includes 30 handouts describing games, activities, and materials which can be applied to math lab situations. Another handout describes a set of 11 "math lab kits" (collections of resource ideas built around a related set of manipulative materials), and a final handout explains the design and use of math lab activity cards. (DT)

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CMM:006:05(ISU) TEAM LEADER NOTES: MATH LAB KITS AND ACTIVITIES 1 of 1

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Objectives:

This module is quite an extensive collection of ideas and materials which will provide the Intern opportunity to plan, modify, and implement what is termed as MATH LAB ACTIVITY CARDS, GUIDESHEETS, and individualizing-types of materials. The Intern should attempt to utilize many ideas dealt with in Module CMM:006:04(ISU) "The Math Lab Approach" in working with the items in this Module.

This module attempts to involve the Intern in an extensive "APPLICATION EXPERIENCE," with a vast array of materials. Of course, the Intern should adapt or design his/her applications only to the degree that the educational setting and other conditions allow. Attempts may start with a MATH INTEREST CENTER, table or corner, BUT it is hoped that the Intern's efforts will be with individuals and small groups and NOT so much with whole class instruction. Some ACTIVITIES, games, CARDS, GUIDESHEETS, etc. may be used with a whole class, BUT this would be the exception rather than the rule. The materials and suggestions in this module should be viewed as supplemental to whole class instruction rather than supplanting that effort. However, the activities and ideas included in this module should not be viewed primarily as enrichment! They can replace the majority of memorized drill-type activities with number facts and basic computation! An activity is much more significant when learners work through a variety of approaches and applications of facts and operations.

Pre-Post-Assessments:

There are three forms which constitute reports of the Intern's demonstrations. It is important that these forms have attached to them samples and descriptions which support the Intern's attempts to carry them out.

Instructional Activities:

These are of three main types:

1. Handouts describing games, activities, or materials. These are sources of many excellent LAB ACTIVITY CARDS OR GUIDESHEETS.

2. MATH LAB KITS which are sources of materials and ideas built around a MATH AID or set of devices. These KITS form excellent sources for developing MATH ACTIVITY CARDS OR GUIDESHEETS.

3. MATH LAB ACTIVITY CARDS. These are suggested CARDS which include both a "Teaching Notes" portion and the actual MATH LAB ACTIVITY CARD that a learner might use. These are only samples and should not be viewed in any sense as a complete set. The Intern is urged to CUT these apart and organize both the "Teaching Notes" and the CARDS for easy access and use. The CARD format is only one way that such individualized materials might be displayed, thus the Intern should be encouraged to ADAPT and MODIFY these materials as he/she sees fit.

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Final Suggestion:

The Intern will have to SCAN READ a volume of material in this module in order to make sense of it. If a perusal survey is made of the various materials, this should help in orienting the Intern to the vast array of resources included. This module could almost be viewed as a culminating attempt to bring together objective which were to be demonstrated in Modules CMM:006:01 through CMM:006:04(ISU).

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Objectives:

At the completion of Module Study, the Intern will be able to. . .

1. (Performance) MODIFY many existing math content and learning activities consistent with the principles of the MATH LAB APPROACH. (REFER to "Principles of Math Lab Approach.")

2. (Performance) ANALYZE the content and the structural form of math materials or activities which would provide for MATH LAB Applications. (Refer to "Principles of Math Lab Approach.)

3. (Performance) SYNTHESIZE a plan for DEVELOPING and IMPLEMENTING USE of an extensive set of RESOURCE MATERIALS and ACTIVITIES for MATH LAB Applications. (Utilize the sheet: "Planning a Math Lab Approach")

4. (Performance and Consequence) INITIATE the plan, which was synthesized in Objective 3, and REPORT to what extent the activity DEMONSTRATES that Learner's are responding at successively higher levels of the category within the AFFECTIVE DOMAIN called "RESPONDING"--from "Acquiesence in Responding" to "Satisfaction in Response." (UTILIZE the form: "Application of LAB Experience")

Pre-Requisites:

Completion or concurrent study of modules CMM:006:01(ISU) through CMM:006:04(ISU).

Pre-Assessments:

The objectives in this module involves Performances of the Intern which include MODIFYING materials and activities in order to implement MATH LAB Applications; therefore, cannot strictly be Pre-Assessed. If an Intern attempts to satisfy the Pre-Assessment, the following Forms should be completed and submitted to the Team Leader and Instructor:

1. "Principles of Math Lab Approach." (attach samples and descriptions which support your efforts for Objectives 1 and 2.)

2. "Planning a MATH LAB Approach." (attach evidences of items a, b, and c.)

3. "Application of LAB Experience" (This is an application of the planning of a MATH LAB Experience and should include data reporting Learner involvement.)

Instructional Activities:

The Handouts and references included with this Module are to be viewed as resources for your use. You are encouraged to TRY OUT a number of the activities and materials by DOING them yourself before using them with learners.

1. REFER to Dumas: Math Activities for Child Involvement for many suggested materials and activities which lend themselves to a MATH LAB Approach. (CAUTION: Many of the items described should be adapted and modified to provide more inductive, exploratory experiences than indicated. Vary the rules, modify the

Instructional Activities: (Cont.)

1. (Cont.)
game play, etc. You are urged to modify activities so that they rely less on memory and recall and more on reasoning and experimenting. The latter will allow for many Learners to approach an activity at appropriate levels of Knowledge and Skill.)

2. REFER to KIDD: Laboratory Approach. . . for many excellent GUIDESHEETS (Learner and Teacher's Guides) and ideas for starting MATH LAB ideas. The GUIDESHEETS may be used as is, modified, or serve only as suggestions.

3. READ and TRY OUT ideas and materials contained in the following Handouts: (some suggestions are noted for each set of Handouts.)

   a. JACOBS: "If the Hands Can Do it the Head Can Follow" (Suggests many ideas for MATH LAB ACTIVITY CARDS, the role the Teacher plays when servicing Math Lab Activity, and suggestions for applying several of the principles of MATH LAB Approach. The EGG CARTON COMPUTER is one example of an Inductive Lab Activity filled with Learner Hands-On Activity.)

   b. "Graphing Pictures"
   (Many MATH LAB CARDS and Activities can stem from use of graph or coordinate ideas. Examples in this Handout involve graphing in all 4 quadrants, thus necessitating use of negative numbers. This can be simplified so that it is not necessary to use but one quandrant--No. II which involves only positive pairs of numbers. Series of MATH LAB CARDS are possible involving graphing of pictures.)

   c. MAY: "Frame Arithmetic: The Art of Asking Questions."
   (Frame Arithmetic lends itself to developing many MATH LAB ACTIVITY CARDS. The Rules of Frame Arithmetic can be taught on one LAB CARD and then other subsequent CARDS will operate within those same Rules unless the CARD suggests differently. Frame Arithmetic assists learners in questioning and analyzing before solving a problem, thus teaching them HOW TO SOLVE PROBLEMS in at least one way. Patterns and basic number properties are particularly stressed.)

   d. MAY: "Nomographs: Making Plus and Minus Easy"
   (This Handout will be more fully explored within MATH LAB KIT #4 on NUMBER LINES. Nomographs lend themselves to MATH LAB ACTIVITY CARDS because they involve arranging number lines in such a way that patterns of numbers are stressed. The Basic operations of addition and subtraction are particularly displayed and verifiable by using the nomograph. Nomographs can become excellent AIDS for helping Learners Verify facts which they are attempting to learn.)

   e. "How Children Learn Mathematics with Cuisinaire Rods"
   (The Cuisinaire Rods is dealt with in much more detail within the MATH LAB KIT #5 on Cuisinaire Rods and in the Davidson: "Photo/text Guide: Using the Cuisinaire Rods" which is available in the ISU Bookstore. This Handout provides basic introduction to the use of the RODS and lends itself to developing MATH LAB ACTIVITY CARDS. Pages 4-5 emphasize the Philosophy of Active Teaching which is promoted by the use of the RODS as well as many other LAB ACTIVITIES.)

Instructional Activities: (Cont.)

3. (Cont.)

f. "The RODS as a Model for Mathematical Relationships."
(Excellent Handout showing some of the ways to utilize the RODS with older learners in teaching Rational Numbers and structure of the number system.)

g. BROWN: "Applying 'Madam I'm Adam' to Mathematics: A Discovery Project."
(Palindromes as introduced in this brief Handout present some rather new and interesting discoveries about numbers. Any expression--words or numbers--which reads the same forwards and backwards is palindromic. A number of MATH LAB ACTIVITY CARDS could be developed utilizing this relationship. The interesting thing about palindromes is that some will evolve in 1 step while others involve several steps:

1 Step:

```
  12
 +21  (12 Reversed)
 ---
  33  (Palindromic)
```

(TRY OUT several possible Palindromics to see how many steps it takes. Learners will enjoy MATH LAB CARDS developed around this concept.)

6 Steps:

```
   97
   79
 -----
  176     (1 step)
  671
 -----
  847     (2 steps)
  748
 -----
 1695     (3 steps)
 5961
 -----
 7656     (4 steps)
 6567
 -----
14223     (5 steps)
32241
 -----
46464     (6 stpes)
=====(Palindromic)
```

h. "Teaching Tools: How to Use them in Math."
(This Handout emphasizes CREATIVE USES OF MATH AIDS AND DEVICES, but includes tools not commonly utilized in math: Overhead projectors, tape recorders, simple teaching machines, etc. Often, MATH LAB ACTIVITY CARDS can specify the use of special equipment or processes. Learners of most any age can be easily and safely taught to use overheads, tape recorders, especially cassette, and other A/V devices. You will need to adapt many of the suggestions in this Handout in order to utilize these tools in MATH LAB ACTIVITY CARDS.)

i. STONE: "Could Amanda Be a Turtle? and other Mathematical Mysteries."
(This Handout emphasizes the designing of MATH Activities which develop thinking, problem-solving, and Learner expression rather than drill. Such stories as "Could Amanda Be a Turtle?" could become an ACTIVITY CARD.)

j. BLOOME: "Bottle Caps Go To School"
(This Handout presents a number of ideas for the use of BOTTLE CAPS in counting experiences, SET ideas, and for learning of various basic number principles. A number of games involving Bottle Caps lend themselves to ACTIVITY CARDS.)

k. MAY: "Variations of Mathematical Themes."
(Suggestions of game-type ideas that lend themselves to MATH LAB CARDS.)

Instructional Activities: (Cont.)

3. (Cont.)

l. RANUCCI: "What's in a Name?"
(Name Arithmetic is an activity which could easily adapt to a MATH LAB CARD.)

m. "Chess and Mathematics"
(Chess as well as many other thought and logic games can lend themselves to MATH LAB Activities. This Handout identifies a way to mark blocks which reveals the basic play rules for different chess pieces. The actual game rules and procedures must be obtained elsewhere--possibly from a learner who has tried it.)

n. "Kinetic Energy and the Storage of Data"
(This Handout illustrates a science-math activity which could be an excellent small group type of MATH LAB ACTIVITY. It emphasizes exploration of the scientific concepts involved with kinetic energy of bodies by utilizing collection and interpretation of data obtained from first hand experimenting by Learners.)

o. MAY: "Search for Patterns Helps Children Think for Themselves."
(A number of suggestions in this Handout lend themselves to MATH LAB ACTIVITY CARDS.)

p. Trotter: "Five 'Nontrivial' Number Games"
(Strategy Games are important additions to math activities and lend themselves to MATH LAB ACTIVITY CARDS.)

q. "Math Lab Activity: ARROWMATH"
(Arrowmath involves some basic rules--which learners can modify to suit their purposes--and can utilize number charts, calendars, and other displays of number arrays. Several suggestions employed in this Handout will lend themselves to Development of MATH LAB ACTIVITY CARDS.)

r. "Numbers Have Many Names"
(This Handout allows exploration of many patterns of numbers involved in one number system. Many MATH LAB CARDS could be developed from such simple procedures as "Pick A Date")

s. ARNSDORF: "A Game for Reviewing Basic Facts of Arithmetic."
(This Handout describes a Card Game which can easily be made and placed on a math lab table. Variations of the game could be created, thus making several games from this basic game.)

t. Cantlon, Homan, Stone: "A Student-Constructed Game for Drill with Integers"
(This Handout describes "hands-on" activities which lend themselves to development of games or MATH LAB ACTIVITY CARDS. The basic game "Popsicle-Stick-Game" described is a SMALL GROUP Activity.)

u. "Tuf Game"
(This Handout describes a commercial Game which can easily be made from small blocks made of wood or other material. A copy of the game is available on loan from the Teacher Corps Office if desired. This game is excellent for diagnosing where learners are in mastery of basic facts and operations on numbers. It involves variations which can be utilized by individuals, small groups, or teams of learners. Requires no Teacher supervision after initial ground rules for play have been established.)

Instructional Activities: (Cont.)

3. (Cont.)

v. "Game of Freebles"
(This game is simple but provides opportunities for much data gathering for learners who wish to seek "why" it works. This is a simple TOPOLOGY game.)

w. "Three-in-a-Row"
(This game lends itself to MATH LAB CARDS. Many variations could be developed by learners.)

x. "Math Lab Activity--Patterns"
(This material is typical of items from the MATH WORKSHOP materials developed by Robert Wirtz and distributed by Encyclopedia Britannica Corporation. Pattern work can be exciting to learners as they discover much about the structure of number systems.)

y. "Math Lab Activity--Squares"
(There are two Handouts entitled this way and both involve some interesting ideas for MATH LAB CARDS.)

z. "Math Lab Activity--Assumptions"
(Helping learners retrieve HOW THEY ATTACK AND SOLVE PROBLEMS are crucial activities which are too often not allowed or not encouraged in math classes. These kinds of activities can provide teachers with much diagnostic information!)

ab. "Classification"
(Processes such as classification must be specifically taught and extended with learners. Several suggestions in this handout lend themselves to MATH LAB CARDS to develop "classification skills.")

cd. "Flow Charting"
(This is an excellent way for learners and teachers to plan an activity or lesson. It will also help to analyze the steps, operations, and explanations--rationale--for math processes.)

ef. "Observation"
(Process skill that needs much development in math.)

gh. "Story Problems"
("Math Headline Stories" is a technique which lends itself to development of many MATH LAB CARDS.

4. REFER to Handout entitled "MATH LAB KITS." This material describes a set of 11 kits--math materials and activities which are built around a related set of math aids. Many LAB ACTIVITY CARDS can be developed from the ideas within the kits. The kits are mainly collections of resource ideas for use by the teacher. The kits are:
#1--INCH BLOCKS
#2--PAPY'S MINI'COMPUTER
#3--GEO-BOARDS
#4--NUMBER LINES
#5--CUISINAIRE RODS
#6--ATTRIBUTE BLOCKS
#7--PLACE VALUE DEVICES
#8--FRACTION KIT
#9--GAMES
#10-ARRAY BOARDS
#11-VENN RINGS

Instructional Activities: (Cont.)

5. REFER to the Handouts "MATH LAB ACTIVITY CARDS." The one handout explains what they are and their intended use. In order for the CARDS to be most useful, the two parts must be CUT APART--"Teaching Notes" and the "Math Lab Activity Card" itself. The "Teaching notes" portion of the CARDS are intended to alert the Intern (or Teacher) to the USES and PREPARATIONS necessary for learners when trying out the CARDS. The Intern is urged to develop other ACTIVITY CARDS to expand the CARDS available to areas they are or will be working with Learners. It is suggested that the "Teaching Notes" portion of the CARDS be placed in a loose leaf binder (4" x 8" size) and tabbed so that the Intern can refer quickly to the same number that appears on the ACTIVITY CARD. For instance, there is a set of "Teaching Notes" for CARDS G-13 through G-21, thus the Intern would be able to refer to these "Teaching Notes" as Learners attempted to work through CARDS G-13, G-14, G-15, G-16, G-17, G-18, G-19, G-20 and G-21.

6. The Intern is encouraged to DESIGN and CARRY OUT any other Instructional Activities which are related to the Objectives of this Module.

Post-Assessment:

The Intern should COMPLETE and SUBMIT the following to the Team Leader and then Instructor:

1. The COMPLETED form: "Principles of Math Lab Approach" including SAMPLES and DESCRIPTIONS which support your efforts for OBJECTIVES 1 and 2.

2. The COMPLETED form: "Planning a MATH LAB Approach" including EVIDENCES of items a, b, and c. (OBJECTIVE 3)

3. The COMPLETED form: "Application of LAB EXPERIENCE" which utilizes the "Plan" developed in Objective 3. Include DATA which reports how learners were involved.

Remediation:

Confer with the Team Leader and Instructor to determine need to re-do or extend any efforts to complete demonstration of the objectives of this module.

CMM:006:05(ISU) Application of Lab Experience 1 of 1

INTERN:__________________________________

NOTE: The three levels within the Affective Domain Category with which the Intern is concerned are:

2.0 RESPONDING:
- 2.1: Acquiesence in Responding.
- 2.2: Willingness to Respond.
- 2.3: Satisfaction in Response.

A re-reading of the following Handouts would be helpful in planning this demonstration of Objective 4:

TTP:004:03(ISU) "A Condensed Version of the Affective Domain of the Taxonomy of Educational Objectives" pp. 2-4.
TTP:004:03(ISU) KIBLER: "Examples of Infinities and Examples of Direct Objects" page 1
TTP:004:03(ISU) GRONLUND: "General Instructional Objectives and Behavioral Terms--Affective Domain" page 1.

YOUR TASK:

As you Implement your plan developed under Objective 3, mark during a period of at least 3 weeks what Learner Repsonses are. Use "A" to show "Acquiesence in Responding", "W" to show "Willingness to Respond," and "S" to show "Satisfaction in Response." ADD notes to support the ratings you assign:

Learners:	1st week	2nd week	3rd week
1.			
2.			
3.			
4.			
5.			
6.			
7.			
8.			
9.			
10.			
11.			
12.			

CMM:006:05(ISU) PRINCIPLES OF MATH LAB APPROACH 1 of 1

_____1. Content and Learning Activities are arranged and made available to Learners as "problems to be solved" or "areas to explore."

_____2. The method is largely inductive but follows rules or procedures which will aid the Learners in discovering patterns and relationships and in forming concepts and generalizations.

_____3. The Teacher serves primarily as a stimulation-initiator and a resource to Learners by arranging environments, posing problems, initiating and probing Learners' explorations, and by gathering evidences of Learners' progress in Knowledges and skills.

_____4. The Teacher does a minimum of explaining or other Teacher-oriented presentation or demonstration. The Teacher's efforts primarily are designed to stimulate and provide support for Learners' Direct, Hands-On activity with materials and ideas.

_____5. Learners work individually, in pairs, or in small groups more than they do in a large class structure.

_____6. Materials and activities are available for many simultaneous studies of content areas. These are frequently added to or modified in some way to provide for different levels of Learner Activity.

_____7. Frequently, Learner explorations take place in natural settings which necessitate Learners being outside the classroom. (Measuring of distances, and solving problems involving objects outside the classroom are two examples.)

_____8. Primarily empahsizes "What is Learner Ready For?" Thus involves gathering and utilizing diagnostic information by the Teacher and Learners. (This implies that not all Learners are at similar stages of readiness for any particular math content, process, computation, material, or activity.)

_____9. Emphasizes "Learning How to Learn" for Learners by encouraging divergent approaches, explanations, and representations by Learners of problems or content being explored.

(NOTE: To ANALYZE AND MODIFY materials and Activities, the Intern should SUBMIT how each principle will be applied. Attach any sample materials and descriptions which tend to illustrate the application of the principles.

In synthesizing a plan for a Math Lab Approach, describe how you will provide for the following:

_____1. How you will organize and arrange your materials--MATH ACTIVITY CARDS, GUIDESHEETS, etc.

_____2. How you will display and manage materials and activities with a group of learners.

_____3. How you will encourage learners' Hands-On activities and how you will assure a minimum of Teacher-Oriented activity

_____4. How you will attempt to gather, store, and interpret information which you gain about learners in order to put such information to diagnostic use.

_____5. How you will structure activity so as to emphasize individual and small group work rather than whole class activity.

_____6. How you will utilize materials and resources outside the classroom for learner use.

In Synthesizing a plan for DEVELOPING and IMPLEMENTING USE of an extensive set of RESOURCE materials and Activities for MATH LAB Applications, provide for the above 6 aspects. Your plan should be submitted in WRITING and include:

___a. How you will provide for the above 6 aspects.

___b. A sketch of how you will physically utilize classroom space.

___c. A sample of how you will structure items 1 and 4.

IF THE HANDS CAN DO IT THE HEAD CAN FOLLOW*

Israel Jacobs

Maureen looked down at the egg cartons. Tentatively she separated the singles, then she took the three sets of four out of the group and put them aside. Hesitatingly she continued the process of separating the cartons until the whole set of egg cartons was apart. She pursed her lips and stared down at the disassembled cartons. A pencil and paper lay nearby to record comments. She reread the activity card:

Materials: One set of nested egg cartons (one twelve, two sixes, three fours, six twos, and twelve singles).
Report any pattern you find
Comment: What do you think of all this?

While Maureen was absorbed in her activity, the other three girls in the group kept up a running commentary:

"These hold only six eggs."
"And two of these make one whole."
"What about these little ones?"
"How many are there of them?"
"One, two...how many of them have you got? Eleven...no, twelve of them."
"Twelve makes one whole, too!"

Maureen sucked the eraser of her pencil a moment and then wrote: "They all fit into each other." She paused and then added, "It has to do with fractions."

The pencil and paper were put aside. The four girls started to rearrange the pieces, fitting them into various groupings, nesting them into each other to make one whole.

Maureen and her group were involved in an experimental project at Intermediate School 27 (Anning S. Prall) on Staten Island, N.Y. It was a mathematics laboratory program that started on May 21, 1971 and continued every Friday thereafter until the end of June. Some one hundred twenty children in the seventh grade were involved. Within each class, the children were divided into teams of four or five of their own choosing. The teams were seated around double desks that had been turned so that the youngsters could face each other. Except for two pilot programs none of these children had ever participated in such an activity before.

A mathematics laboratory approach is not new. In England, the Nuffield Mathematics Project was set up in 1964. The Madison Project promotes an individualized laboratory approach. The Arithmetic Teacher has included reports of a number of mathematics laboratory projects. The purpose of a mathematics laboratory is to involve the hands of the youngsters--an approach that will, we hope, lead to the understanding of concepts.

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*The Arithmetic Teacher. November. 1972, pp. 571-577

The program at Prall was not funded. All games were constructed by the author with the help of the youngsters. Many of the activities were culled from the mathematics laboratory course for teachers conducted by Professor Judith Jacobs at Richmond College, Staten Island, N. Y. Suggestions for other activities came from the pages of the Arithmetic Teacher and from various textbooks.

In the mathematics laboratory, the role of the teacher was to hand out prepared activity cards that provided instructions for the pupils to follow and to give limited assistance if it was needed. At first the teacher's function was to walk around, smile, and to each anguished question answer, "I don't know. What does the card say?" If a group was completely stalled, then the attitude of the teacher changed. A leading question from the teacher usually precipitated meaningful action.

Maureen and her group were playing with egg cartons that the children had collected and brought in. Twenty groups were involved in this activity. A majority of the groups did not relate this game to fractions immediately or discover any kind of relationships among the various pieces beyond discovering the parts of twelve. One group turned the cartons into a tossing game. They used the 12-section piece as a receptacle and tossed the singles at it. The winner was the one who scored the highest number of "ins." Other groups used the cartons as erector sets to build various shapes.

The second activity with egg cartons included specific questions:

How does the smallest carton relate to the largest carton?
How does the next smallest carton relate to the largest carton?
What other relationships do you find?

The response to this activity was positive. Answers to specific questions were well within the students' experiences. It was on this activity that a group of advanced students came up with some interesting results.

Two teams in the advanced class had moved their desks together and formed a group of eight. Their work area was covered with egg cartons, bottle caps, and sheets of scribbled paper. A debate was in progress. Sue placed bottle caps in the egg receptacles, counted them, made notations on a sheet of paper and handed the paper over to Glenn. Seven heads bent over as they checked it. They looked up and Glenn said, "It works."

"What works?" I asked.

"Our computer. A computer to multiply fractions!" said Sue.

"How does it operate?"

A single 12 egg carton was set out and Sue explained. "Suppose you want to show a fraction of 5/6 on the computer. Place a bottle cap in ten of the openings....."

I held up a hand. "Let's backtrack just a little bit. Ten? Why ten?"

She sorted through the sheets on the desk and pulled out a diagram. (See figure 1.)

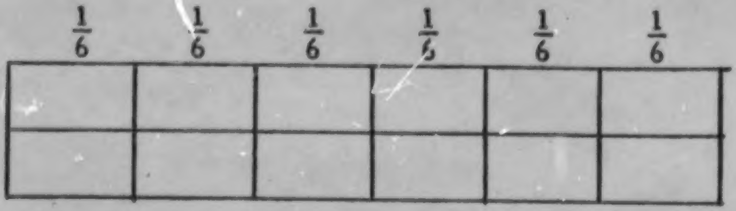


Figure 1

"Each double up and down row....."
"Vertical."
"Each vertical row is 1/6 of the egg carton. If we want 5/6," she explained, "we place a large bottle cap in ten of the holes. Now our computer reads 5/6."

"But," I objected, "you have ten caps in the computer. How does that come out 5/6?"

"Look," said Glenn, pointing to another diagram. (See figure 2.)

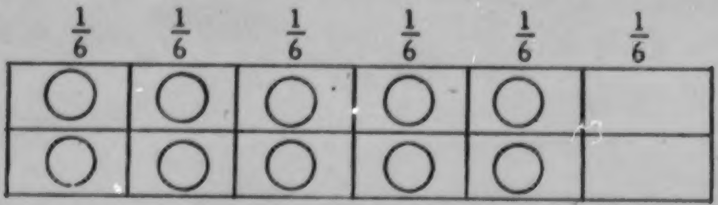


Figure 2

"This computer is a 12 hole computer. So we double the 5/6. That's why this computer shows ten bottle caps in a 12-hole carton. Suppose we want to multiply $\frac{1}{2}$ times 5/6. Now we place small bottle caps over the large bottle caps, but only on half of them like this." (See figure 3.)

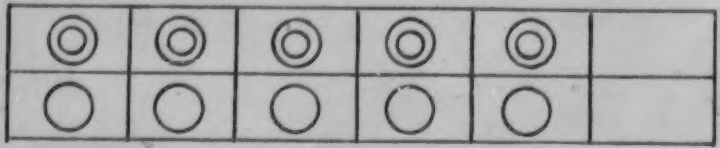


Figure 3

"Hmm. How would you show 1/7 on your computer?"
"Easy!" they chorused.

A 2-section egg carton was added to the original set and figure 4 was the resulting diagram.

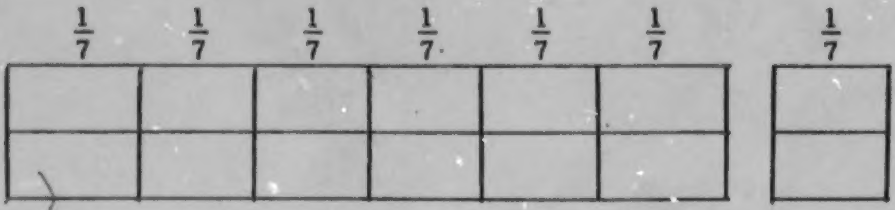


Figure 4

"What would $\frac{1}{2}$ of 3/7 be?" I asked.
"Let me show it now," said Sue, "I found it first."
"Go right ahead," said Glenn, "but I figured out the 14-hole computer to start with."

Sue went on to explain. "Since all the holes in the egg cartons total fourteen we call this a 14-hole computer. Now for 3/7, all I have to do is put six large caps in the openings, because each vertical row has to be filled completely." She went on, "For $\frac{1}{2}$ of 3/7, all we do is put three small caps over the large ones, and the result--3/14!" (See figure 5.)

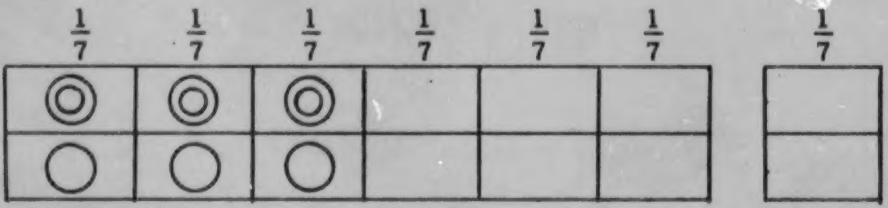


Figure 5

"That's all very nice," I conceded. "The egg cartons are conveniently set up in sets of pairs. Suppose you want your computer to show 1/3 of 3/7?" With that I walked away.

In a little while, Sue and Glenn signaled frantically. The egg cartons had been arranged as in the diagram in figure 6.

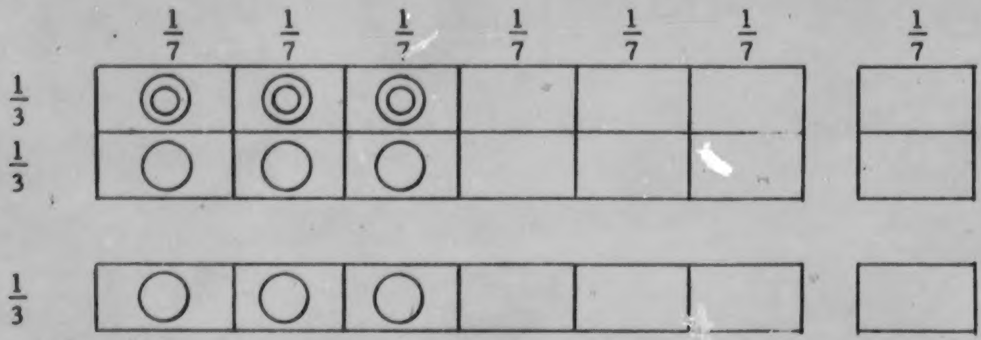


Figure 6

"Now," explained Glenn Triumphantly, "we have a 21-hole computer."
"Why 21?"
"Because," explained Sue, "if you want 1/3 of 3/7 we need a "whole" into which both 3 and 7 can go evenly. And that is 21."

Glenn interposed. "In this 21-hole computer, 3/7 means nine large caps in nine holes. To take 1/3 of 3/7, we put three small caps over the three large ones, and the computer now reads 3/21."

"Very good! Write it up." I walked away.

It is possible that either Sue or Glenn may have come across fraction diagrams before and applied this knowledge to the egg cartons. Whether this exercise in fraction multiplication was a discovery or just reinforcement, it was a worthwhile experience for the whole group.

From the front of the room, left, a boy was screaming, "I won! I won!"

He was in a group of four playing the spinner fraction game. Two of this same group had been active in constructing the game. Round cardboards, eight and one-half inches in diameter and in various colors, had been cut up into pie slices of 1/2, 1/4, 1/8, 1/16, and 1/32. (See figure 7.) In cutting up these slices, the youngsters quickly learned that it took two 1/32-pie slices to make one 1/16-pie slice.

1/2
1/2
1/4
1/4
1/4
1/4

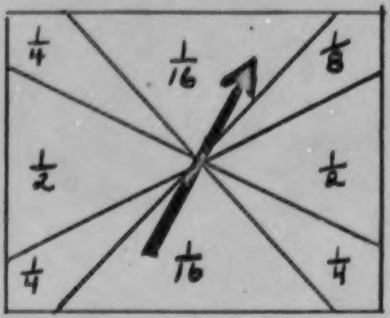
(Red) (Blue)

1/8
1/8
1/8
1/8
1/8
1/8
1/8
1/8
1/16

(Green) (White)

(Yellow) Figure 7 Black Playing Board

The spinner was made in the school shop. The sections on the spinner board itself are labeled 1/2, 1/4, 1/8, 1/16, as shown in figure 8.

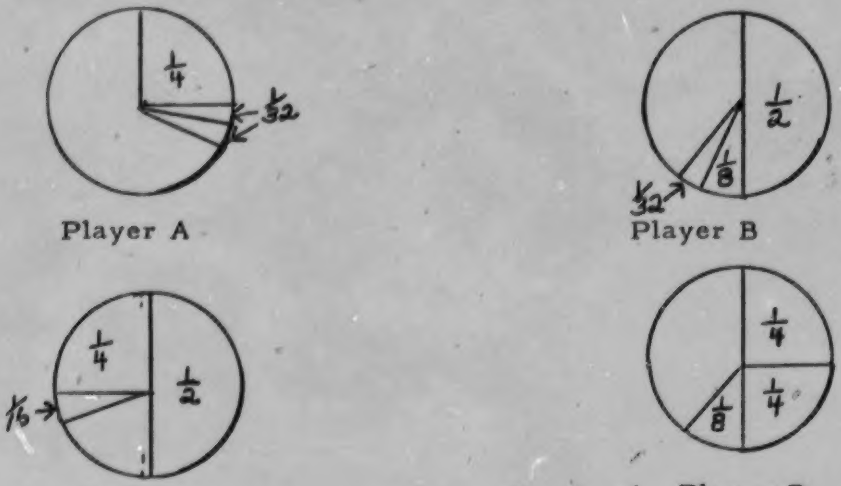


SPINNER

Figure 8

The wooden arrow, pegged loosely to the board, is spun by each player in turn. Each player has to select from the general pile of pie slices a piece corresponding to the fraction the arrow on the spinner points to, and he places the piece on his playing board (the black one). To win, all fractions on the playing board must total up to one whole. If the spinner selects a fraction that does not fit on his board, then the player must return an equal amount to the general pile.

Figure 9 shows the playing boards of four players (A, B, C and D) in progress. On his next turn, C gets 1/4 on the spinner. Since this cannot be used on C's board, he has to return a 1/4 slice to the general pile so that C's board then looks like figure 10.



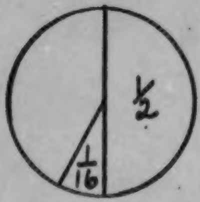
Player A

Player B

Player C

Player D

Figure 9



C's Board after he lost the "1/4" slice.
Figure 10

After he lost the 1/4-pie slice, C had just two slices left, 1/2 and 1/16.

Suppose C had had no 1/4-slice on his board. He would have had to change his 1/2 to two 1/4s and give up one of them. Such changes were checked carefully by each player. The students learned to add and subtract these fractions without the use of pencil and paper.

Spinner fractions was the most popular game, and all the youngsters were anxious to play with it. It was the spinner that intrigued them. At one time when the spinner was out of action for a while, cards with fractions marked on them were substituted. The cards were not as much fun for the students as the spinner.

The second activity with the spinner fraction game added something new. It was not enough to tabulate the number of wins and the ways in which a game could be won. Now the goal was to determine which fraction on the board came up most frequently. The students were to try 100, 500 and 1,000 spins at a time.

One group in each class worked on this activity. All of the groups that were involved concluded that the "winningest" fraction was 1/2. One boy simply wrote "The more we spin the more it's 1/2." No one group came up with an specific reason why this was so, although one of the boys did comment that he could make a better shaped arrow!

"Does the shape of the arrow have anything to do with 1/2 coming up most often?" I asked.

He did not know.

Another activity card read as follows:

SCALE MEASUREMENT
Materials: road map, string, 12-inch ruler.
Use string to follow along the main roads between selected cities.
How many miles are there between the selected cities?

Interest in this game varied. Some groups were absorbed all period. They were the ones who had worked out the meaning of the scale at the bottom corner of the map. Other groups, those who could not gifure out the meaning of the mileage scale, tired of this game very quickly and went to something else. Still others just sat and gossiped until the scale idea was explained.

Another activity card gave the following instructions:

MEASUREMENT IN INCHES, FEET AND YARDS

Materials: piece of string, 12-inch ruler.
Measure as many objects in this room as possible.
Record results in feet and yards. Compare results with other groups for accuracy.

The group that showed the most interest in this activity (and had the most fun) was in a class of low achievers. They had a wonderful time measuring each other's heights, including the height of the teacher. They measured desks, chairs, the length of the floor, and even the length of a girl's blond hair. They did discover that different measurers came up with different results for the same object measured. Errors were quickly recognized. The boy who translated 72 inches into 5 feet was corrected by everyone in his group. Generally this activity evoked such enthusiasm as to involve everyone in the group.

From the youngsters' point of view, the mathematics laboratory was a huge success. There was always laughter, and at times, the joyous noise level was so high that the place did not sound like a classroom at all. No one was cracking the obvious learning whip. There were no tests to take, no homework to worry about, no notes to copy. It was a period of freedom. On Fridays the students sought the author out on the stairways, in the hallways, and in the lunchroom; or they stopped by his room to ask only one question, "Are we having math lab today?"

Two obvious questions can be raised. What did this experiment prove? Is a mathematics laboratory a worthwhile program of learning on an extended basis?

The experiment was implemented toward the end of the school year, and it was of short duration. Generally, the activities were haphazard; the pupils could try one game or another, visit, or gossip. Learning was casual, self-oriented, and untested; but it was there, whether we call it discovery or reinforcement. Whenever an activity card was deliberately vague and allowed the youngsters a choice of direction, the usual response at first was a dead stop. As the program progressed, the children caught on to the new idea; they were to proceed on their own.

The strict regimen in mathematics is all too familiar. It does not reach all of the youngsters. The phrase under-achiever was coined for that group of students who find mathematics too difficult to comprehend. A mathematics laboratory could be programmed once a week and it could be coordinated with the week's lessons. It should make more use of manipulative activities than pencil and paper activities. In our experimental mathematics laboratory program, the youngsters who physically worked with inches, feet, millimeters, and centimeters learned about measurement more quickly (and had a great deal of fun with it) that those who learned with pencil and paper exercises and table memorization. On an extended basis, learning with the hands can be fun, and it can become more meaningful to the student.

The role of the teacher has to be more than to just stand aside and hope that Johnny will make a great discovery or get started on his own. Usually Johnny does neither. It is in a laboratory setting, with proper guidance, that Johnny can learn to get started on his own and perhaps go in a direction of his own choosing and still have fun doing it. But he must be placed in such a position again and again before he learns that he is allowed to wander off on his own and that this wandering has value.

During the final Friday program, some of the youngsters brought in wood dice they had made in shop.

"Why can't we use these in the games?"
"You can. How true are they?"
"They're pretty good. I made them."
"How would you test them out?"
"Use them, I guess."
"Would you like to try it out and bring in a report?"
"How many times would I have to roll them?"
"100, 500, maybe 1,000 times."
"Boy! It'll take all summer."
"Not as long as that. Is it worth a try?"
"O.K., 1,000 times!"

GRAPHING PICTURES

Coordinate graphing and experience on the coordinate plane is very useful mathematical knowledge. Informal work in this activity leads to graphing of equations on the secondary level and is formally explored in Analytic Geometry. Experiencing numbers in geometric situations adds another dimension to the mathematical insights of children. Coordinate graphing is not only sound mathematically, but fun!

The coordinate plane has four quadrants to define possible location of pairs of positive or negative numbers. Children must become familiar with placement of negative numbers on a number line before proceeding to graphing pictures.

Begin with the number line, only call the zero position "home" and the positive direction "east" (Figure 1).

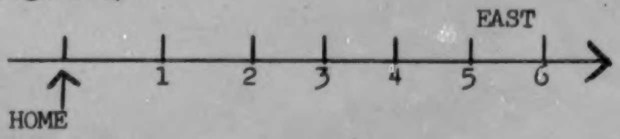


Figure 1:

Ask children to find: 2 east, 5 east, etc. Pose the question of a series of jumps: Begin at home, go 2 east and 3 more east and then 4 west. Where are you? (1 east) Extend the line to the west of home (Figure 2).

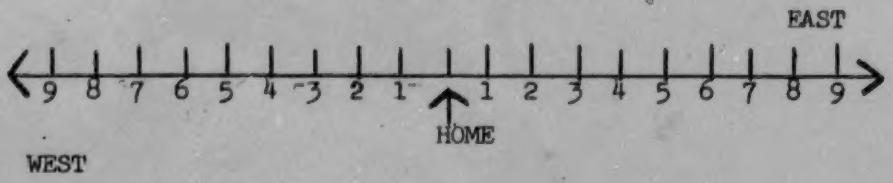


Figure 2:

Find where you would land if you begin at home, go 5 east, 6 west, and 1 east (home). Continue with moves on the east-west number line until children are thoroughly familiar with position on this horizontal line. The next step in developing the coordinate place is to develop distance north and south of home. This is done by drawing a vertical number line through home (Figure 3).

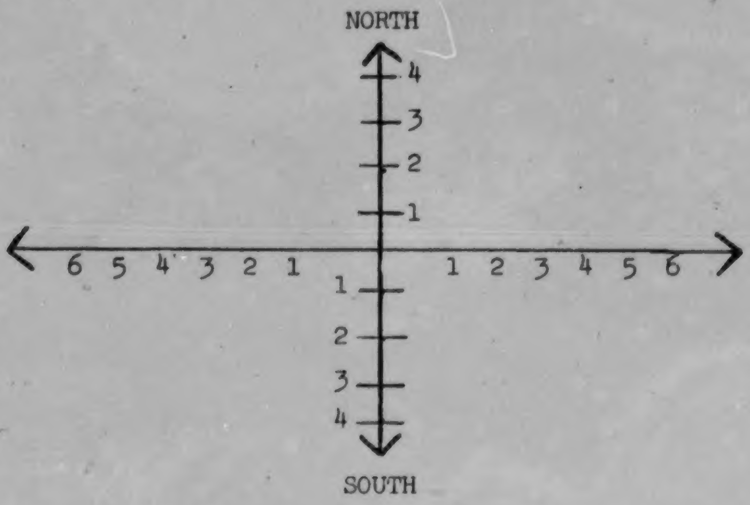


Figure 3:

GRAPHING PICTURES

Develop the north-south vertical number line by a similar method to the east-west one. If you begin at home, go 4 north and 2 more north, then go 8 south, where would you land? (2 south). Play many such games on the north-south line until children can locate positions.

Now, the next obvious extension is to ask the children, where would you land if you went 5 east and 3 north (think of blocks)? We would land in the upper right quadrant of the plane. It is useful to trace in the intersecting lines for the plane. You have now developed the coordinate plane (Figure 4).

The next step is to remove home and replace it with zero as the origin and to eliminate the east, west, north, south (only if your children are ready!!) and develop conventional notation for any given pair of coordinate numbers. The east-west (x-axis) is given first. For example, the pair (5,3) means 5 east and 3 north, whereas the pair (3,5) means 3 east and 5 north. West and south directions become negative numbers as shown. Note points as shown by coordinate pairs in parentheses (Figure 5).

A large graph of coordinate space should be drawn on the chalkboard or projected by an overhead projector. Using the space, a game can be developed where one side gives the coordinates and the other side plots them on the graphing space. Keep score to see who can best plot using both positive and negative number pairs. Once an understanding of negative numbers is achieved, children can be given graph paper and sets of coordinates to be plotted and connected in sequence to outline pictures.

We have reproduced the coordinate plane full size as you might reproduce it on a ditto master (Illustration 1). Prepare many of these forms for use in producing different pictures. When preparing the master copy, you will not, of course, draw in the shape of the Key, for this is the student's job as he follows the directions. The directions may be printed on the ditto (each ditto sheet is then limited to the production of only 1 specific picture) or printed on the board for different pictures.

Try some of the following suggestions.

After children have tried producing these pictures according to given instructions, challenge them to prepare instructions for different pictures. The class can then try to follow the instructions to see if they are accurate. Illustration 2, 3, 4, 5, 6.

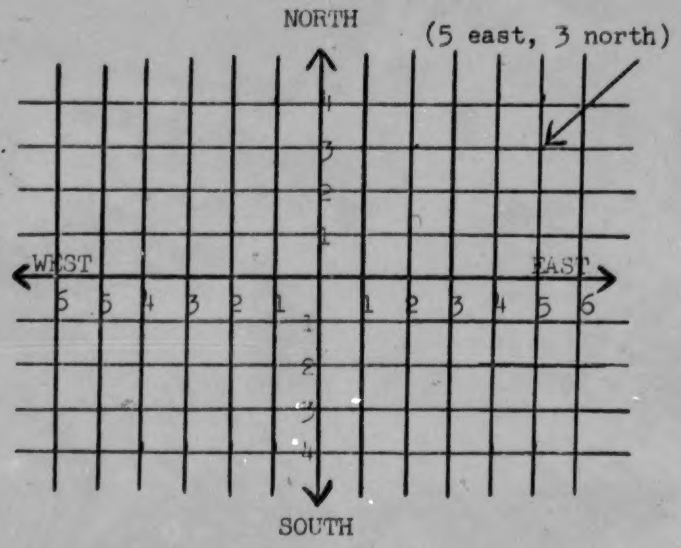
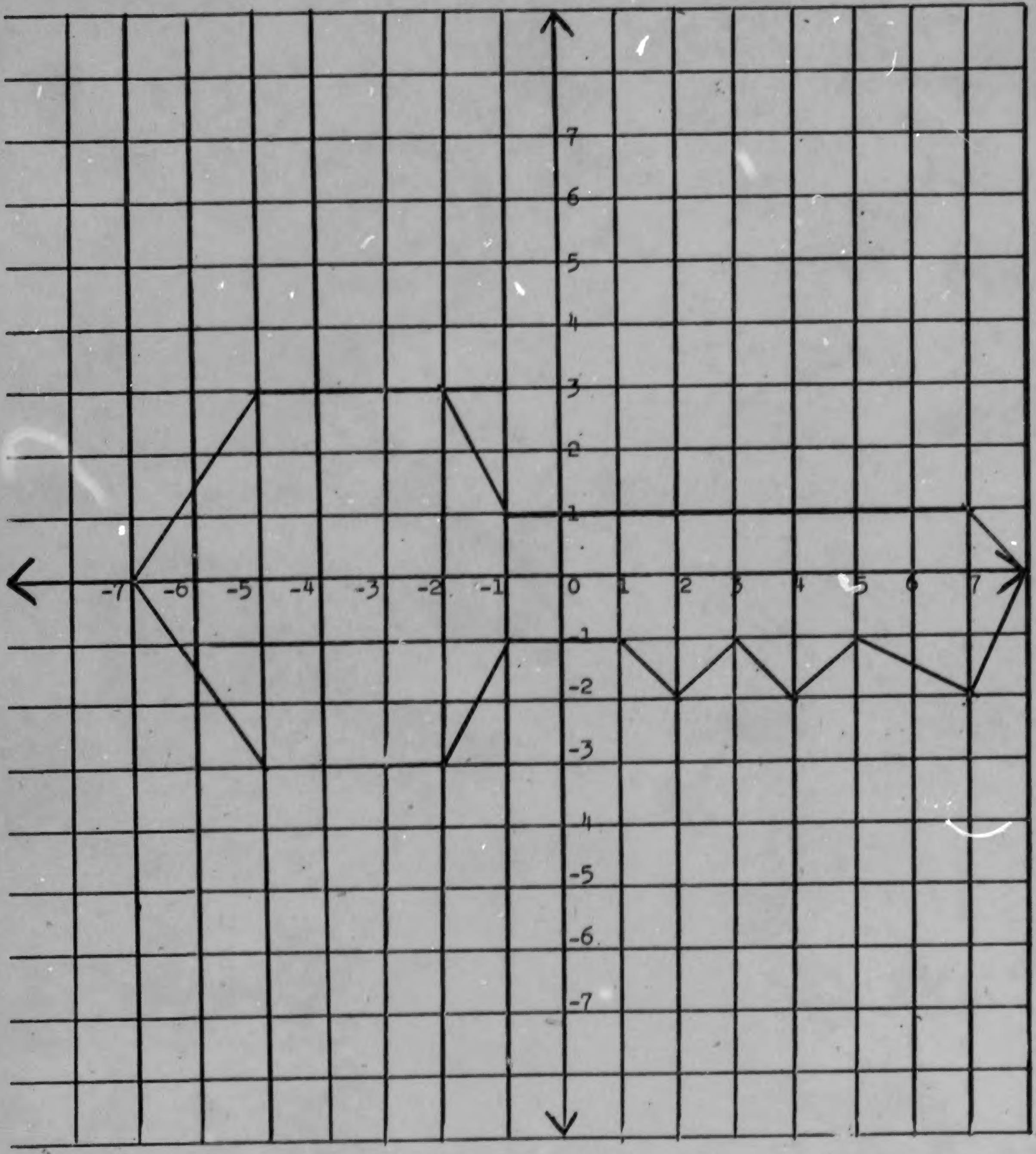


Figure 4

GRAPHING PICTURES



KEY
(-7,0), (-5-3), (-2-3), (-1-1), (1,-1), (2,-2), (3,-1)
(4,-2), (5,-1), (7,-2), (8,0), (7,1), (-1,1), (-2,3), (-5,3)

Illustration 1:

GRAPHING PICTURES

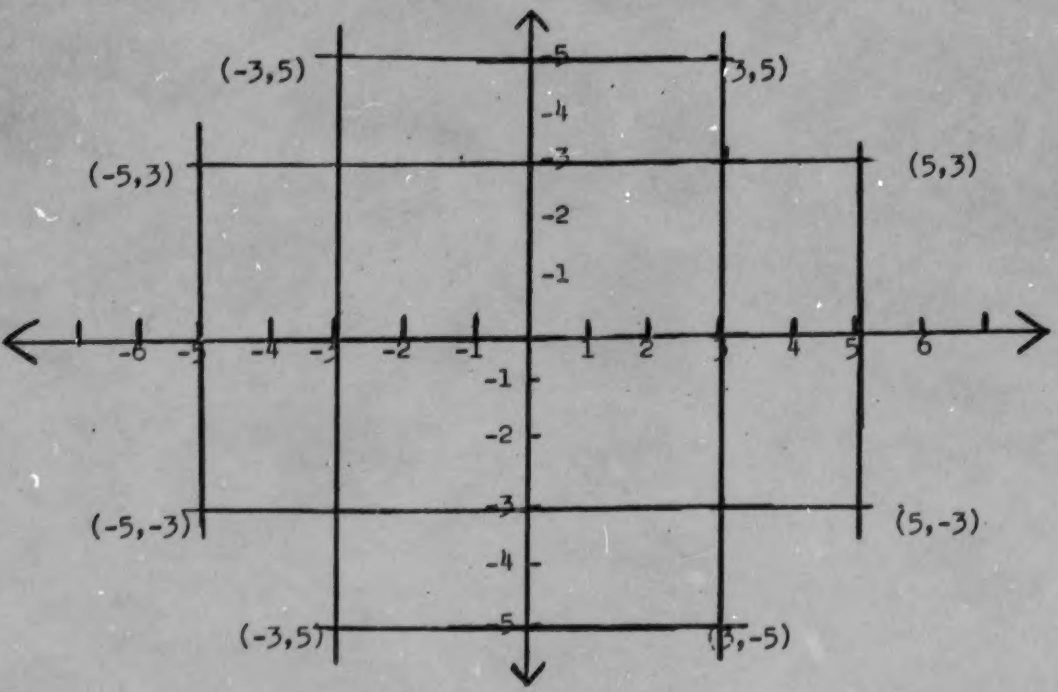

(-3,5)
(3,5)
(-5,3)
(5,3)
(-5,-3)
(5,-3)
(-3,5)
(3,-5)
5
4
3
2
1
-1
-2
-3
-4
5
-6
-5
-4
-3
-2
-1
1
2
3
4
5
6

Figure 5:

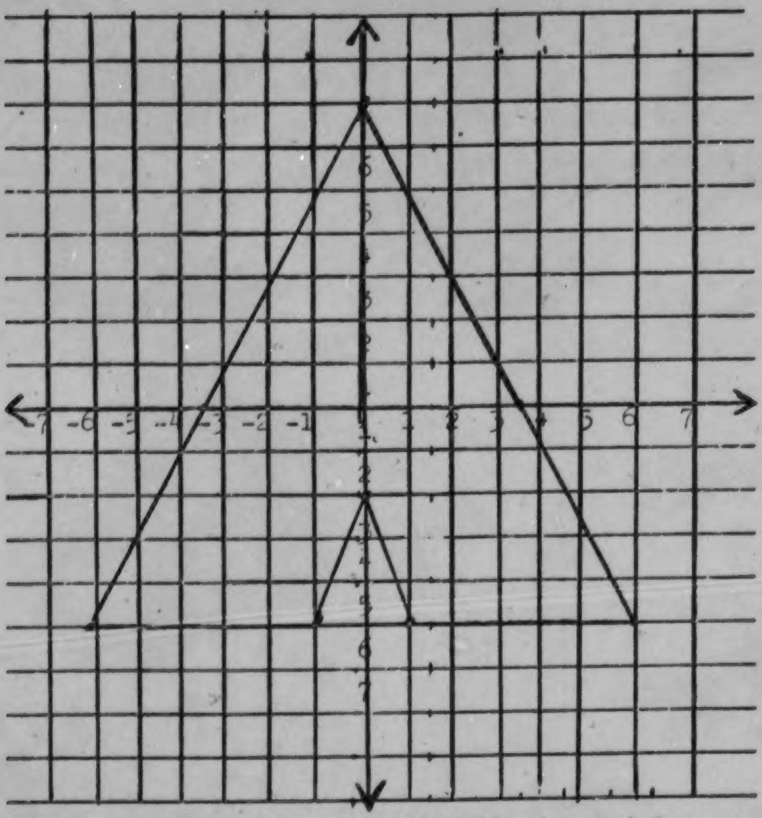
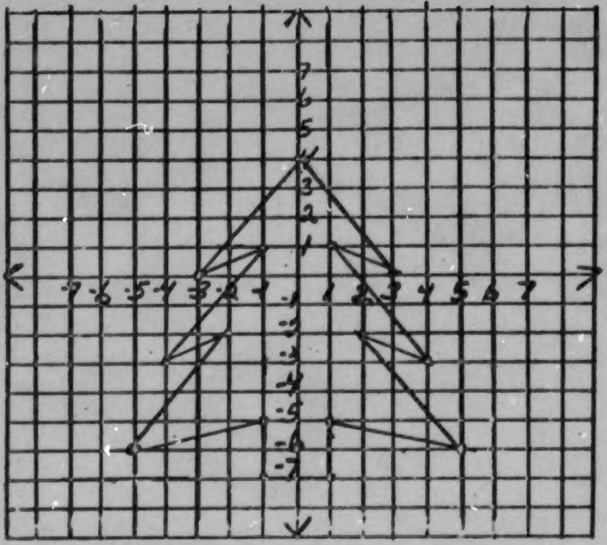

-7
-6
-5
-4
-3
-2
-1
0
1
2
3
4
5
6
7

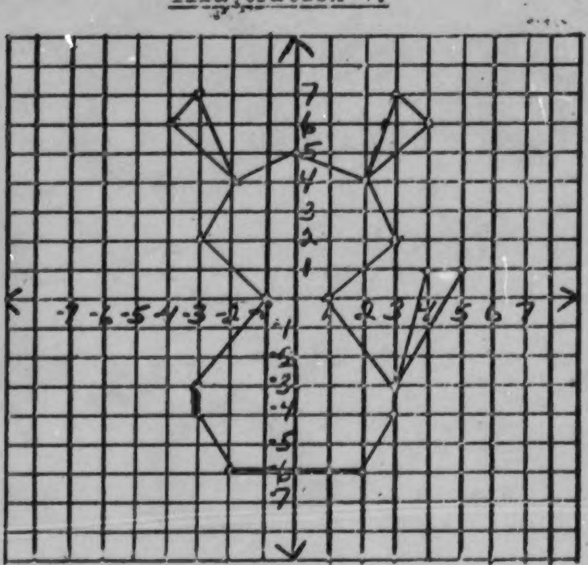
Illustration 2: TEXT: Locate and connect the following points:
(0,-2), (1,-5), (6,-5), (0,9), (-6,-5), (-1,-5)


7
6
5
4
3
2
1
-7 -6 -5 -4 -3 -2 -1
1 2 3 4 5 6 7
-1
-2
-3
-4
-5
-6
-7

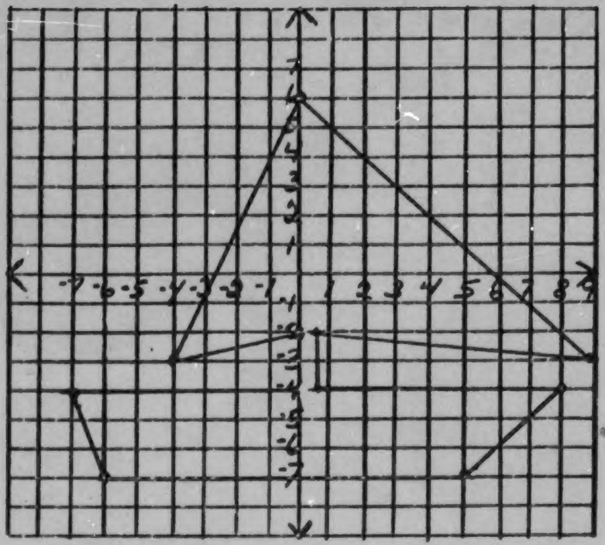
Christmas Tree
(1,-7), (1,-5), (5,-6), (2,-2), (4,-3)
(1,1), (3,0), (0,4), (-3,0), (-1,1),
(-4,-3), (-2,-2), (-5,-6), (-1,-5),
(-1,-7)

Illustration 3:

Illustration 4:


7
6
5
4
3
2
1
-7 -6 -5 -4 -3 -2 -1
1 2 3 4 5 6 7
-1
-2
-3
-4
-5
-6
-7

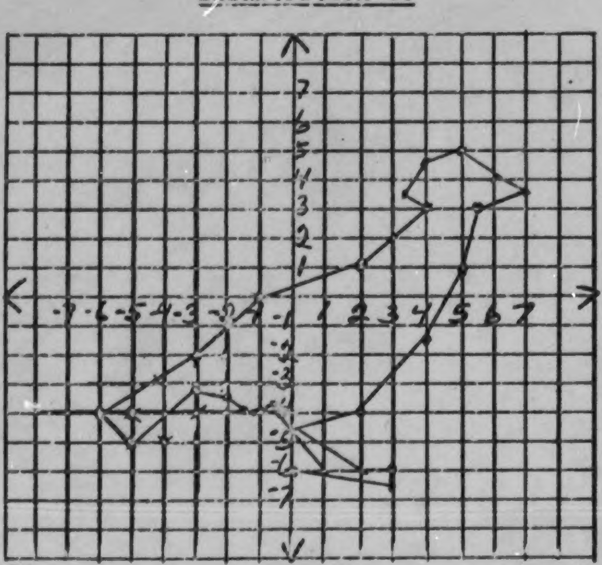
Mouse
(2,-6), (3,-4), (3,-3), (5,1), (4,1), (3,-3)
(2,-1), (1,0), (3,2), (2,4), (4,6), (3,7),
(2,4), (0,5), (-2,4), (-3,7), (-4,6),
(-2,4), (-3,2), (-1,0), (-2,-1), (-3,-3),
(-3,-4), (-2,-6)


7
6
5
4
3
2
1
-7 -6 -5 -4 -3 -2 -1
1 2 3 4 5 6 7 8 9
-1
-2
-3
-4
-5
-6
-7

Sailboat
(5,-7), (8,-4), ($\frac{1}{2}$,-4), ($\frac{1}{2}$,-2), (9,-3),
(0,6), (-4,-3), (0,-2), (0,-4), (-7,-4),
(-6,-7)

Illustration 5:

Illustration 6:


7
6
5
4
3
2
1
-7 -6 -5 -4 -3 -2 -1
1 2 3 4 5 6 7
-1
-2
-3
-4
-5
-6
-7

Pigeon
(0,-6), (2-6$\frac{1}{2}$), (2,-6), (1,-6), (0,-4), (2,-3),
(4,-1$\frac{1}{2}$), (5,1), (5$\frac{1}{2}$,3), ([illegible],3$\frac{1}{2}$), (6,4),
(5$\frac{1}{2}$,[illegible]), (5,5), (4,4$\frac{1}{2}$), (3$\frac{1}{2}$,3$\frac{1}{2}$), (4,3)
(2,1), (-1,0), (-3,-2), (-6,-4), (-5,-5),
(-3,-3), (-2,-3$\frac{1}{2}$), (-1$\frac{1}{2}$,-4), (-$\frac{1}{2}$,-3$\frac{1}{2}$), ($\frac{1}{2}$,-6)

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CMM: 006: 19 (ISU)

FRAME ARITHMETIC: THE ART OF ASKING QUESTIONS*

Here's the way to get your youngsters really thinking about the questions posed by mathematical problems.

BASIC MATH a chore? Not any more. Today we use diagrams to illustrate basic math operations. It is a different approach, but most effective. It's called frame arithmetic and besides taking the drudgery out of basic math operations, it teaches students to question and analyze before solving a mathematical problem.

Frame arithmetic provides a challenge. The student must understand what each frame stands for before he can solve the problem. The frames in each mathematical sentence represent numbers which are variables--just like the letters in algebra. The shape itself is immaterial. You can use squares, triangles or even clouds to make things a little more interesting--any geometrical shape will do as long as you remember these two rules:

Rule 1: If the frames are alike in a sentence, the same number must be used to replace the frames:

$$\square + \square = 8$$

Since the frames are alike, there is only one number which can make the above sentence true. The number is 4.

Rule 2: If the frames are different in a sentence, either different numbers or the same number can be used as replacements for the frames:

$$\square + \triangle = 8$$

If we limit the replacement set (numbers that can be used to replace the frame) to the set of whole numbers, there are nine different solutions to this sentence. The answers can be shown in a table:

□	△
8	0
0	8
7	1
1	7
6	2
2	6
5	3
3	5
4	4

In the primary grades, the teacher usually designates the set of replacements as the set of whole numbers. In the upper grades, however, the students can use the set of positive rational numbers (fractions) as the replacement set and then obtain an infinite number of answers.

What numbers will make this sentence true?

$$\square + \triangle - \square = 6$$

---

*From: Grade Teacher, 25-28; By Lola May (1970)

Remember, whatever number replaces the first square must replace the other square as well, while the number replacing the triangle can be the same or different. Students will start to give such answers as: 5 replaces the squares and 6 replaces the triangle; or 7 replaces the squares and 6 replaces the triangle. After three or four solutions they will see a pattern developing:

□	△
5	6
7	6
10	6

etc.

The pattern is that there can be an infinite number of solutions--you can use any number to replace the squares, but you must use 6 to replace the triangle. Just think about this for a moment and you'll see the underlying logic: When you subtract a number from itself the result is always zero, and when you add a number to zero the sum is always the same as the number added to zero.

Now we'll move on to another facet of frame arithmetic. What number will make this sentence true?

□ x △ = △ x □

Since whatever number you use to replace the square or triangle on one side must be used for the similar frame on the other side, you'll get answers like 3 X 8 = 8 X 3 or 9 X 3 = 3 X 9. There are many solutions since this is an example of the commutative property of multiplication. This means that even if you change the order of the factors, the product remains the same.

The above sentence problem is also known as an identity. In an identity, any corresponding set of numbers can be used to make the sentence true. It's a good idea to have your middle-graders make up other frame sentences which are identities. These are the sort of thing they should come up with:

□ + △ = △ + □
The commutative property of addition.

□ + O = □
The identity element of addition.

□ x 1 = □
The identity element of multiplication.

□ ÷ 1 = □
□ ÷ □ = 1
The property of one.

After your students grasp the basics of frame arithmetic, you can start to limit the replacement set. This will be even more challenging to your students. Take this sentence for example:

E⃞ + E⃤ = 12

The E inside the two frames means that the replacement set for both the square and the triangle is the set of even numbers. The solutions can be listed in a table like this:

$\boxed{E}$	△
0	12
2	10
4	8
6	6
8	4
10	2
12	0

Another set can be given in which 0 is written inside the frames so that the students will have to use the set of odd numbers as replacements:

$$\boxed{O} + \triangle = 12$$

$\boxed{O}$	△
1	11
3	9
5	7
7	5
9	3
11	1

One more sentence makes this series of exercises complete. It reads:

$$\boxed{E} + \triangle = 12$$

Your students will probably assume that the E and the 0 are the set of even numbers for the square and the set of odd numbers for the triangle. However, as they try to work this out, they'll see it's impossible to arrive at an even sum with these two frames.

This exercise points out the rule of adding odds and evens: When you add an odd to an odd, or an even to an even, the sum is even--but when you add an odd to an even or an even to an odd, the sum is always odd.

Similar exercises can be tried where P is written inside the frame, and the replacement set is the set of prime numbers. It's important to vary these problems so the challenge to your students remains fresh.

Won't it be great to have the whole class contributing various correct answers when you wirte on the blackboard--square plus triangle plus cloud equals 12? Frame arithmetic is so much more fun than doing arithmetic.in columns. But more important--you will be challenging your students to really think about math problems.

Here's a simple device with a complicated name that enables students to solve problems with a ruler.

Chances are, you've had lettle contact with the field of mathematics known as nomography. It wasn't taught when you were in elementary school; and it's hardly likely that you've seen it mentioned in books on the "new math."

Nevertheless, you owe it to yourself (and your youngsters) to learn more about nomographs--what they are and how they can be constructed. They're ideal for helping students develop skill in reading graphs and using the ruler, But even more important, they force the student to practice that most fundamental step in mathematics--analyzing the problem before attempting to solve it.

Nomographs are calculating charts designed specifically to simplify finding the solutions to mathematical problems. They're based on analytic geometry and are widely used today in engineering, industry and the physical sciences.

Accurate nomographs for addition and subtraction can be constructed by you or your students. There's really nothing very difficult about them. The nomographs we use consist of three scales--the top and bottom scales representing the addends and the middle scale representing the sum in a mathematical sentence.

The intervals between numbers are identical on both addend scales, while on the sum scale the distance between intervals is one-half of the addend scales.

Once you've constructed a nomograph, the rest is child's play. First, analyze the sentence to find out whether the missing number is an addend or a sum. Then place a ruler (or anything with a straight edge) over the known numbers on the appropriate scales, The ruler will automatically cross the missing number on the third scale.

If you're one of those teachers who need proof, take a look at Figure 2. Let's say you want to add 2 and 4. Place your ruler across 2 on the top addend line and 4 on the bottom addend line. If your ruler doesn't cross 6 in the middle (sum line), you'd better get anoth ruler--yours is crooked.

Before you introduce your class to nomographs, though, give the youngsters a little practice in reading different number scales. Have your students fill in the correct numbers in the cirlces (see Figure 1). In order to do this, they'll have to count spaces, by 1's, 10's and 100's. After the children have mastered the use of the number scales, they'll be ready for the next step.

Here, they must differentiate between the addends and the sum. Have them write A for addend and S for sum under the appropriate parts of the mathematical sentence. If the children have analyzed the sentence properly, it should be clear sailing from there on in.

Eventually, you'll want you youngsters to construct their own nomographs. This means that you'd better learn how to construct one yourself. It's every bit as easy as using one.

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*From: Grade Teacher, January 1970, pp. 24-28; By Lola J. May.

Let's try constructing a nomograph with the numbers 0 to 10 on the addend lines. Begin by drawing a straight line. To make things easy, let's make the line 10 inches long. Obviously, the numbers 2 through 9 will be one inch apart.

The bottom addend line in this nomograph is identical to the top line. No problem there, It's only when you start constructing the sum line in the middle that you'll have to do some figuring.

There are two things to remember when you're constructing the sum line:

First, the distance between the number intervals is half that of the addend scales. This means that you'll designate numbers at half-inch intervals.

Second, the ratio of the sum scale to the addend scale is always 2 to 1. Thus, the first number in the sum scale you're constructing--the one that corresponds with the number 1 in the addend scale--would have to be 2, and the last number in the sum scale would have to be 20.

Let's examine Figure 2 and see how this nomograph can be used to solve problems.

In order to solve the sentence 9 + 6 = ?, the student must draw a line from 9 on the first addend scale to 6 on the other addend scale. The line will intersect at 15 on the sum scale.

In the problem 17 - ? = 8, the student draws a line from 8 on the addend scale to 17 on the sum scale, When this line is extended, it passes through the answer, 9, on the other addend scale.

In the third problem, ? - 3 = 12, the student draws a line from the 12 on one addend scale to the 3 on the other addend scale. This line passes through 15 on the sum scale.

Nomographs may be used with larger numbers and with addend lines which aren't identical. The nomograph in Figure 3, for example, is different from the previous nomograph we used--but exactly the same procedure is employed in solving the problems. (In case you're wondering, the 2-to-1 ratio between the sum scale and addend scale still applies even though the addend scales are different. Just take the average of the two scales--in the case of Figure 3, the 85 and the 95--and you'll find that you always come up with a 2-to-1 ratio.)

Nomographs for adding and subtracting fractions can also be easily made. The nomograph in Figure 4 is constructed for use with fractions that have denominators of one-half, one-fourth and one-eighth. Others can be made using fractions that have denominators of thirds and sixths or fifths and tenths or sixths and twelfths.

Nomographs for adding and subtracting decimals can also be constructed (see Figure 5). As these are used, the idea that 10 tenths is the same as 1 should be emphasized.

Encourage your students to construct their own nomographs based on the models here. And get them to make up their own problems for other students to solve, too.

Your students will enjoy using this "trick" device with the strange name. And as they construct their nomographs and work our their own problems the basic operations, the reading of scales and the use of the ruler will all make more sense.

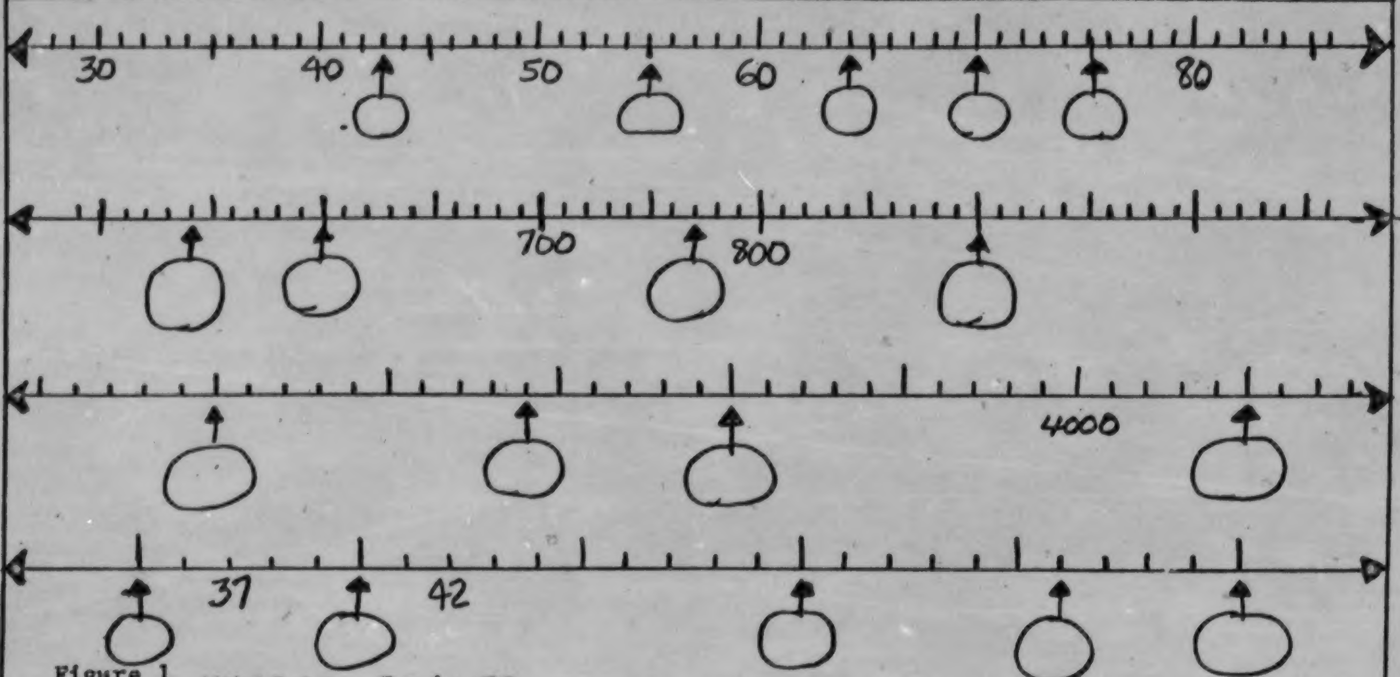
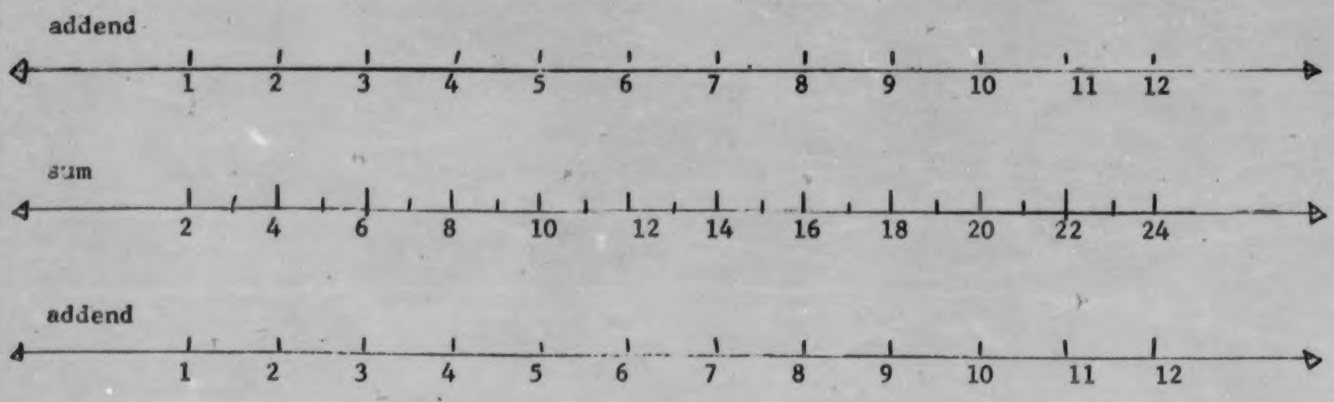

30
40
50
60
80
700
800
4000
37
42

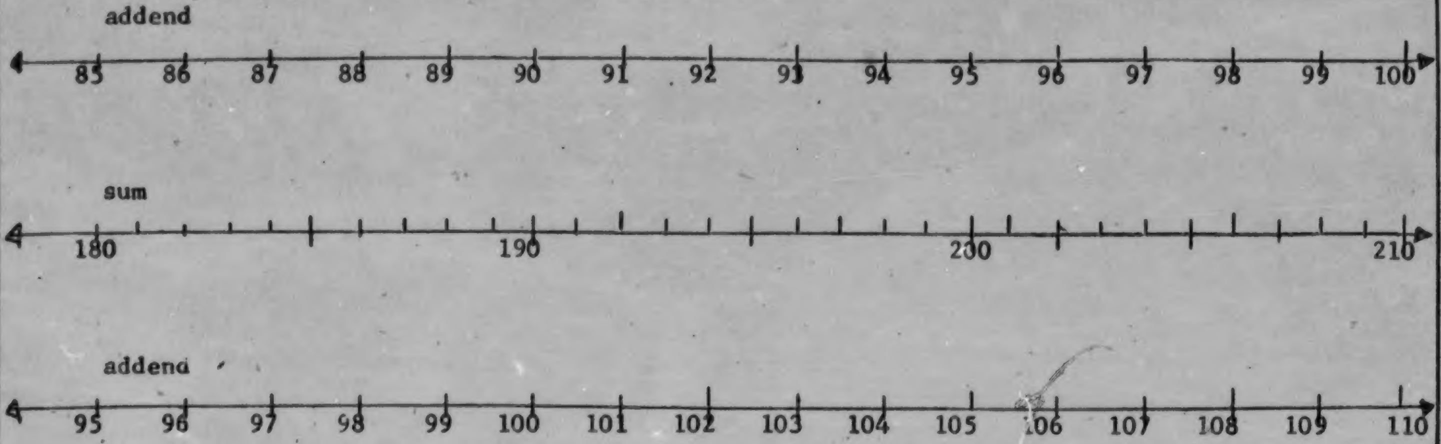
Figure 1 NUMBER SCALES


addend
1
2
3
4
5
6
7
8
9
10
11
12
sum
2
4
6
8
10
12
14
16
18
20
22
24
addend
1
2
3
4
5
6
7
8
9
10
11
12

9 + 6 = ☐ 17 - ☐ = 8 ☐ - 3 = 12

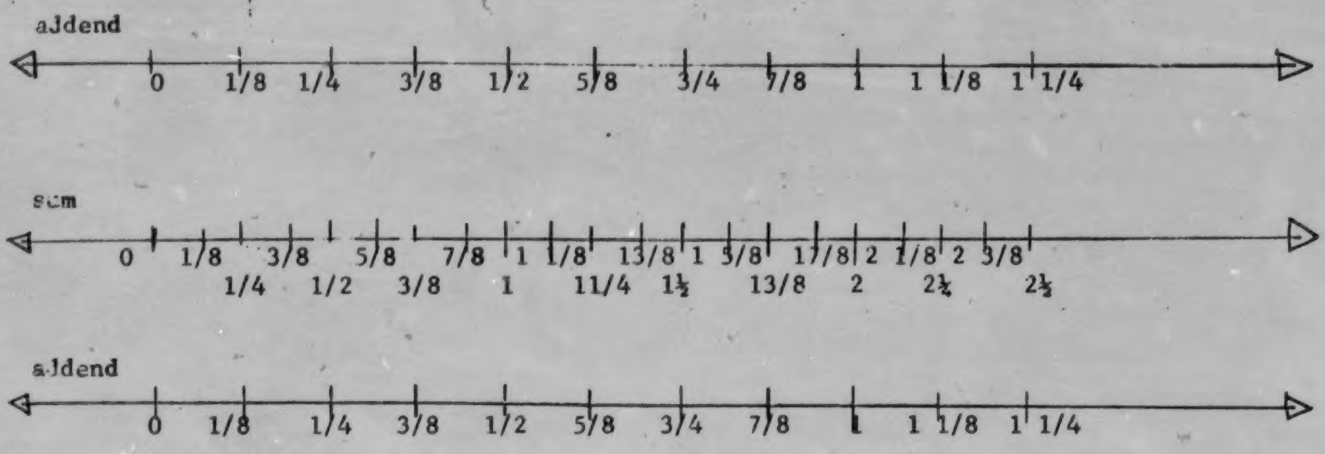
A A S S A A S A A

Figure 2


addend
85
86
87
88
89
90
91
92
93
94
95
96
97
98
99
100
sum
180
190
200
210
addend
95
96
97
98
99
100
101
102
103
104
105
106
107
108
109
110

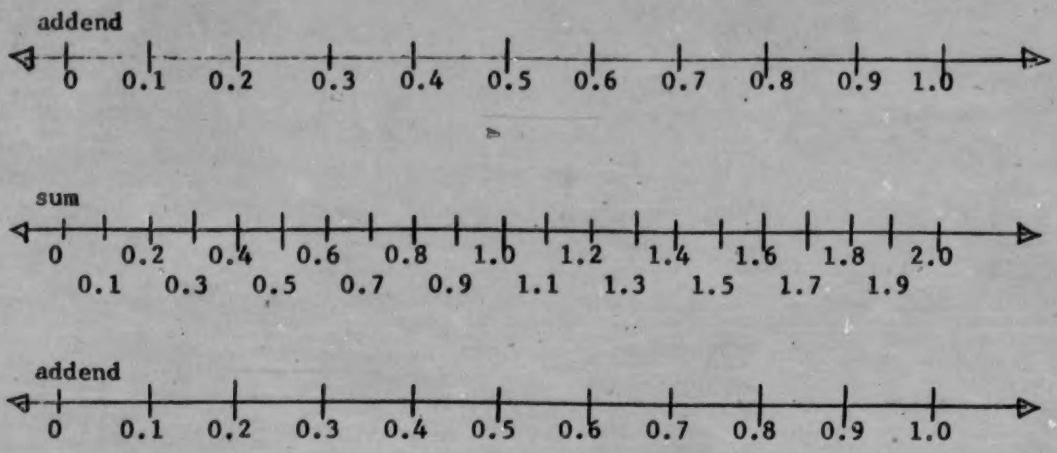
88 + 99 = ☐   191 - ☐ + 97   87 + ☐ = 103
A A S   S A A   A A S

Figure 3


addend
0
1/8
1/4
3/8
1/2
5/8
3/4
7/8
1
1 1/8
1 1/4
sum
0
1/8
3/8
5/8
7/8
1 1/8
13/8
1 5/8
17/8
2 1/8
2 3/8
1/4
1/2
3/8
1
11/4
1½
13/8
2
2¼
2½
addend
0
1/8
1/4
3/8
1/2
5/8
3/4
7/8
1
1 1/8
1 1/4

1/2 + 3/8 = ☐   1 1/4 - 7/8 = ☐   ☐ - 3/4 = 5/8
A A S   S A A   S A A

Figure 4


addend
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
sum
0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0
0.1
0.3
0.5
0.7
0.9
1.1
1.3
1.5
1.7
1.9
addend
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0

0.6 + 0.9 = ☐ 1.4 - ☐ = 0.8 ☐ + 0.3 = 1.1
A A S S A A S A A

Figure 5

How Children Learn Mathematics with Cuisenaire Rods*

cMM

The Cuisenaire-Gattegno approach is based on the essential idea that, by using color funtionally, the gap between the concreteness of everyday life and the abstractness of mathematics is easily bridged, and that children can think mathematically when they are considering obvious relationships exemplified in the colors.

The youngest children do not see the values embodied in the lengths of the rods. Their first impression is of different colors. It follows that they soon separate the heap into groups of the same color. This analytic action is usually followed by an unsystematic attempt at comparison. One of the early discoveries, when a pattern is being built, is the interchangeability of the rods of the same color. This is, of course, a fundamental mathematical structure--the equivalence-relation.

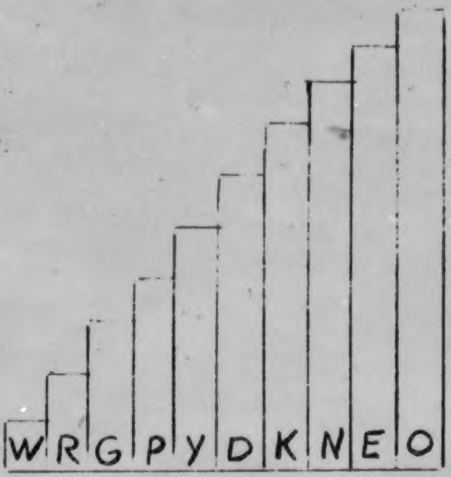
It is enough simply to attempt to build any sort of construction with these rods--as all children do of their own accord as soon as they are given the material--for the possibility to appear of putting two rods end to end to obtain a new length; or, additionally, of replacing two rods end to end with a single rod of equivalent length. This is an algebraic structure. Most mathematical thinking, which is not identified with arithmetic, corresponds in fact to the effect of one such structure upon another.

For the child engaged in work with the rods, the monotonous and uncertain feeling that is associated with counting is transformed into an exciting, intense experience of the active intellectual mind. Through his experience of equality and inequality of rods, equality of length of combined series of rods, and sequences of various kinds, the child becomes familiar with a multitude of arithmetical facts. The experience of the child manipulating sets of numbers in color is that of the true mathematician who exploits to the full the potentialities of one situation and one idea.

The set of rods can be "structured" to exemplify all the arithmetical relationships met with in school life. Any given length, for example, can be constructed by a number of groupings of rods of various lengths. These groupings are called decompositions, or partitions, of that length. Much of the child's early learning activity consists of making such patterns and then describing the facts which have been discovered. Figures 2 and 3 show how decompositions can convey addition, subtraction, multiplication, factors, division, and fractions. Decompositions of odd length introduce the concept of primes and of division with remainder.

Thus, the subdivision of mathematical operations, such as leaving fractions to later grades because they are too "hard," does not exist when the Cuisenaire rods are used. Children move freely back and forth between addition and multiplication, factoring and division, fractions and proportion. Every situation is absorbed as it arises. Every number contains a set of relationships and can be obtained by starting from other numbers.

Figure 1. CUISENAIRE ROD "STAIRCASE"



These are the ten rods (shown slightly reduced), arranged as a "staircase" - a pattern which most children produce spontaneously in their preliminary play.

Children quickly learn to pick up any rod with their eyes closed and tell, by its length, what color it is. In a surprisingly short time they learn the abbreviated letter-names for the colors - W(hite), R(ed), G(reen), P(urple), Y(ellow), D(ark green), (blac)K, (brow)N, (blu)E, and O(range).

Each rod is one white rod longer than the one on its left. Each can be expressed in terms of how many "white rods long" it is. But each rod can also be described in relation to any other rod considered as a unit. Thus, if red is chosen as a measuring rod, the others are easily seen to be W = ½, G = 1½, P = 2, . . . O = 5.

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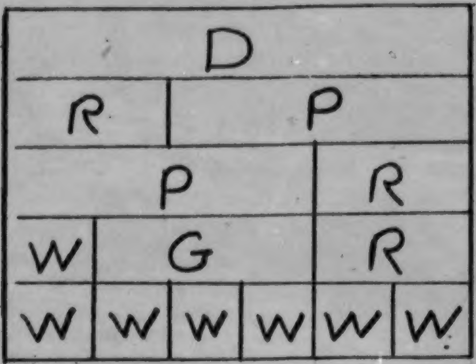
*From Cuisenaire Reporter, Introductory Issue, pp. 7-10.

With this basic knowledge of the materials, children are ready to talk in mathematical language about any arrangements they make with the rods.

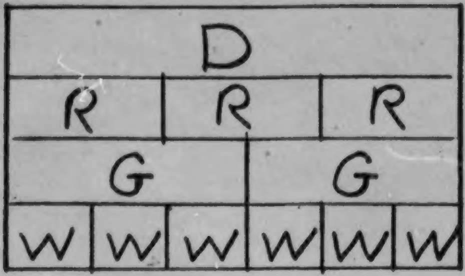
Figure 2. ADDITION AND SUBTRACTION



Color names only	White Rod as Measuring unit	Abstraction
R+P=D	2W+4W=6W	2+4=6
D-P=R	6W-4W=2W	6-4=2
P+R=D	4W+2W=6W	4+2=6
D-R=P	6W-2W=4W	6-2=4
W+G+R=D	1W+3W+2W=6W	1+3+2=6

This is a partial decomposition of the dark green rod. Addition is immediately seen as being commutative. Subtraction is seen not necessarily as "taking away" but as an expression of difference. The child making such a "pattern" becomes intimately acquainted with many ways in which the number 6 can be described.

Figure 3. MULTIPLES, FACTORS, FRACTIONS

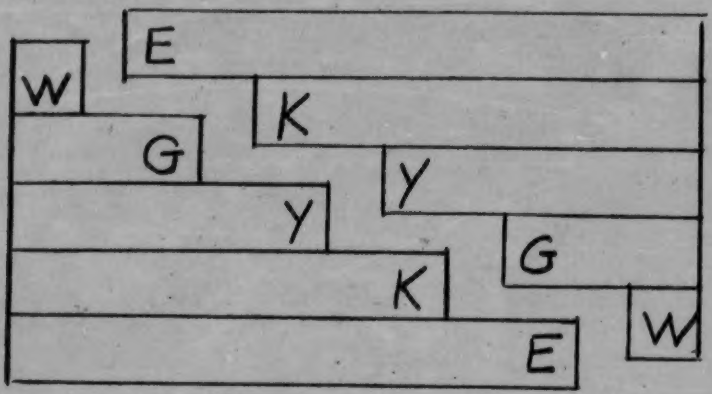


Color names only	White rod as measuring unit	Abstraction
3R=D	3x2W=6W	3x2=6
R=1/3D	2W=1/3x6W	2=1/3x6
2G=D	2x3W=6W	2x3=6
G=½D	3W=½6W	3=½x6
6W=D	6x1W=6W	6x1=6

A child looking at this special form of decomposition--one in which every row contains rods of only one color--can see all the relationships listed above.

In the Cuisenaire-Gattegno system, fractions are introduced almost simultaneously with whole numbers. Given the names--halves, thirds, fourths, etc.--the child finds the concept no more difficult than that of multiplication, of which it is truly the inverse.

Figure 4. ALGEBRAIC RELATIONSHIPS



In this pattern, the difference in length between any two adjacent rods is the length of the red rod. Each "staircase" represents what is called an arithmetic progression. Each rod may be thought of as representing a term in the progression. Adding the lengths of the rods in one of the staircases gives the sum of the lengths in the progression. Readers may wish to examine drawing to see how children can discover that $S=\frac{1}{2}n(a+L)$, where S represents the sum of the progression, a represents the first term, L the last term, and n the number of terms.

It is not remarkable, in the context of the rods, for a first-grader to have created this statement about the number 7:

(1/2 x 12) + (1/2 x 2) = (1/3 x 3) + (1/3 x 18)

This child was simply using mathematical notation to record facts which, from many of his own experiences, he knew to be true.

Written work is introduced as a means of recording observations. Writing should be the servant of thought, and not its master. It is never treated as an end in itself. Mathematical signs, including numerals, are easily and immediately drawn by children of five or six if they are left free to do it as they please, and if they are not expected prematurely to display calligraphic talent. If the teacher puts the work on the board for the children to copy as exercises all that they will understand of writing is that it consists of meticulously reproducing little signs. The writing should be developed only as the child feels the need for notation of the arithmetic he is discovering.

As experiences increase, children meet many aspects of each number they encounter. They learn to express the number 30, for example, as:

30 - 2 x 3 x 5 = 5 x 3 x 2 = 2 x 5 x 3 = 3 x 5 x 2 = 3 x 2 x 5 = 5 x 2 x 3 = 2 x 15 = 15 x 2 = 10 x 3 = 3 x 10 = 5 x 6 = 6 x 5

From this follows quite naturally:

30 ÷ 2 =   30 ÷ 5 =   30 ÷ 3 =
30 ÷ 10 =   1/2 x 30 =   30 ÷ 15 =
1/3 x 30 =   1/6 x 30 =   1/5 x 30 =
1/10 x 30 =   2/15 x 30 =   1/15 x 30 =

Since all the aspects of 30 are met at the same time, the web of relationships found round the number is stronger, better knit, and more readily evoked than the number itself in isolation.

Children who have fully mastered these laws are not slaves to a mechanism they have memorized, nor do they lack ways of arriving at the right answer. An 8-year-old, asked to multiply 18 x 35, gave eight different ways of solving the problem. Among these were:

(10 + 8) x 30 + 5) =
(10 x 30) + (8 x 30) + (5 x 10) + (5 x 8) =
300 + 240 + 50 + 40 = 630

and

$(2^4 + 2) \times (2^5 + 3) =$
$(2^4 \times 2^5) + (3 \times 2^4) + (2 \times 2^5) + (2 \times 3) =$
$2^9 + (3 \times 2^4) + 2^6 + (2 \times 3) =$
$512 + 48 + 64 + 6 = 630$

Another 8-year-old, asked to make up a problem about a real-life situation, produced this:

> Once I went shopping. I saw 48 marbles and bought two-thirds of them. The other day I found 60 marbles in the street but lost half of them. Playing marbles I won 48 and lost 1/8 of them. The next day I won 9 and lost 1/3 of them. How many marbles do I have?

$2/3 \times 48 + 60 - 1/2 \times 60 + 48 - 1/8 \times 48 + 9 - 1/3 \times 9 = 110$

Children who use the Cuisenaire-Gattegno method have an incomparably greater experience of numbers and a more detailed, personal acquaintance with them than is usually the case. The process of doubling and halving, for example, is directly related to the colors of the rods, the orange being twice the yellow, the dark green twice the light green, the brown twice the purple, the purple twice the red. Children soon discover how numbers behave when they are halved and doubled, moving back and forth between one number and the next in the series. The knowledge they acquire through practice of this kind is that there is a crescendo of numbers; that when they have learnt to name the larger ones, these become as real as the small numbers; that besides the slow process of adding the same quantity, and the quicker process of multiplying by the same number, there is the method of doubling which very soon takes them beyond known numbers to the wonders of the infinite.

The material allows as well for a much more articulated experience of space relations (lengths, areas, volumes) than is ordinarily possible. With the rods, a variety of figures can be built, as the children quickly show us, and the problem of finding their areas and volumes is one that may be given to the class at a later stage. Problems from real life situations can, of course, be dealt with by means of the rods. If, for instance, we take a rod to be the total selling price of an article, one of its decompositions can show the proportionate cost of materials, of manufacturing, of distribution, and the profits. A number of problems can be tackled by exploiting to the full a simple situation of this kind.

Thus, with the rods, pupils can learn much more mathematics of a much higher level of abstraction and much faster than hitherto. Without forcing them, they can now, on the whole, reach at seven years of age a level of mathematical experience equivalent to that of eleven-year-olds, and at eleven or twelve the level reached in the higher grades in secondary education. The rods can also be used with profit up to the twelfth grade and even by college students.

With consistent and correct use of the rods, it has been shown: that products can be learned without using tables; that subtraction of any two numbers can be performed without recourse to borrowing; that all processes of arithmetic can be learned at the same time; that fractions are not more difficult than whole numbers and can be used from the start; that a place exists for each aspect of the activities involved in learning and that children can correct themselves and reach a much higher level of mastery than is considered usual; that quick pupils advance further than others without being held back by the slow learners, and find many more meaningful challenges; that slow pupils gain confidence, experience, and hence speed, because they go at their own pace, and that their speed is an increasing function of experience; finally, that the application of mathematics to the real world seems natural to those who have developed within themselves the appropriate structures.

CUISENAIRE'S PHILOSOPHY OF ACTIVE TEACHING

SEEING

Numbers and their multiples are represented by related colors.

The various lengths being of regûlar gradation allows of active use of eyes and hands.

Dimensions and colors constitute a double link between numbers. This classification facilitates the identification of numbers, their groupings, and the discovery of the relationships between them; ensures that they are precisely and firmly fixed in the memory; and prepares the way for mental perception.

DOING

The child's need for action finds an outlet in the spontaneous construction of numerous combinations, freely produced by him and based only upon his awareness of relationships and groupings of numbers. These combinations allow of a great variety of decompositions.

UNDERSTANDING

Seeing and doing lead to conviction and to ease in retaining results. The imagination is stimulated and reckoning becomes automatic.

RECKONING

Through manipulating the rods the child discovers new combinations which increase not only his skill in calculating, but also his interest, experience and knowledge.

VERIFICATION

This is an important phase of the child's experimental work, for he checks his own results and learns to rely on his own criteria for correcting his mistakes.

Thus, by the use of this method of colored numbers:

Each child starts from the beginning and is compelled to re-discover arithmetic for himself, at his own pace and according to his own capacity.

Visual, muscular and tactile images, clearly defined and durable, are created.

Each number acquires and retains its individuality in the numerous combinations and decompositions in which it and its various multiples play a part.

The child is gradually brought to a certain level of abstraction through repeated practice in seeing mentally.

Since it is the child's own thought which takes material form through his own manipulations and with the active intervention of all his senses, colors and dimensions thus being constructively associated, his analytic capacity is developed through his own calculations and his own experience. He acquires without strain mental flexibility and an attitude of objectivity.

Work becomes attractive and interesting, time is saved and the teacher's task is simplified.

The bridge is formed between the early experience gained through play and observation, and the stage of systematic work.

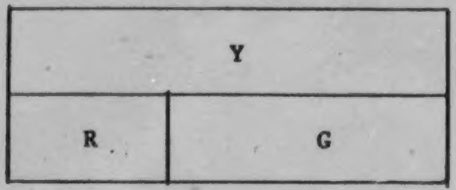
CSM: 006: 05(ISU)

The Rods as a Model for Mathematical Relationships*

Cmm

The structure of the Cuisennaire rods is isomorphic with the structure of the field of of the rational numbers. The rods are considered, therefore, not as an aid to make a theory acceptable, but as a model of what one wants to study. The Cuisenaire rods are in fact a model for the set of rational numbers.

The combination of symbolic statement along with physical example has great advantage over the use of statements represented by ink, chalk, or pencil marks. For example, the statement "2 + 3 = 5" in itself gives no proof of its own validity nor any sense of the many relationships which it implies. But when the white rod is taken to represent a unit length and an arrangement is made such as



one sees, at the same time, the symbolism and the reality it represents--a statement of an abstract truth and an example of the validity of that statement.

Each operation with or upon these objects has its own dynamism and incorporates, at the same time, operations and their inverses as well as computations and results, so that the total structure of a mathematical operation is evident, not as a step-by-step deductive process, but as a physically demonstrated relationship.

When children use the rods they soon recognize three fundamental multivalent structures of modern mathematics:

(a) The relationship of equivalence, that is to say, rods of the same color are the same length (owing to the way they are made), and those of different colors are of different lengths. Moreover, what is known in mathematics as the quotient-set is also noticed when the child represents with any one rod of one color the class of equivalence of all the rods of that color. Clearly, the still more general notions of set and sub-set have been grasped even if they have not been formulated.

(b) The relationships of order, that is to say the fact that, taking two rods a and b at random in the set, the child can say if a equals b or if it differs from b, and when he agrees to describe a as smaller than b (or b larger than a) or vice versa, he links this word to the perception of inequality. Such a comparison between rods is more profoundly structured when the child can combine pairs of inequalities to form a transitive set of propositions: if $a > b$ and $b > c$ then $a > c$, and the like. Not only is the given set of rods ordered, but likewise every sub-set of this set.

(c) The algebraic relationships which result from the introduction of an operation on the set of rods. Every child spontaneously arranges his rods in various ways to produce an extraordinary variety of color patterns. If he becomes aware that two rods end to end replace another rod, or two rods end to end, in length, he explicitly introduces an algebra into the set: an algebra represented by one operation marked by the sign + and called addition such that its obvious properties are $a + b = b + a$, $(a + b) + c = a + (b + c)$, (commutativity and associativity) From $a + b = c$ through a new awareness (and a new notation), $a = c - b$, or $b = c - a$ are drawn. Subtraction is introduced as the inverse operation of addition, and addition appears as the inverse operation of subtraction. If a second operation is introduced and called multiplication, distributive with respect to addition, the algebra is that of a commutative ring; when the inverse of multiplication and a new neutral element are introduced, this transforms the structured set into a field. Multiplication can easily be seen as the iteration (repeated addition) of one and the same rod.

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* From: Cuisenaire Reporter, Introductory Issue, pp. 11-12

As the child can achieve all these fundamental structures when he is given a set of rods, and also since he can recombine them to obtain more special, fertile and rich situations, his mathematical self-education can take place on a realistic basis which conforms with the working of the mind.

All this and much more becomes obvious through the use of the rods. The knowledge of how things are done belongs essentially to the realm of the algebraist, and the early discovery by the pupils that the action of putting rods end to end does not depend on the particular rods that the commutative law is inscribed in the patterns whatever their length, that associativity results from certain equivalences and not from the elements involved, plunges them right from the start into this realm in which the tools because of their range are very powerful and where the mind is unrestricted by particular connections which must be memorized.

When the algebra of the set, that is, the operations introduced and their inverses (when these can be defined), is thoroughly mastered (which takes a few weeks with children of five or six), measurement of the rods one with another will give rise to a new language for the propositions already encountered, and new properties will result from the new structures involved. It is the language of numbers, and we can at this stage embark on the study of the set of rational numbers and their distinctive and important properties.

APPLYING "MADAM I'M ADAM" TO MATHEMATICS: A DISCOVERY PROJECT*

Gerald W. Brown

The quotation "Madam, I'm Adam" is recognized as an expression that reads the same forwards and backwards. Perhaps better known is the quip attributed to Napoleon, "Able was I ere I saw Elba." In language, such expressions are called palindromes.

In mathematics, numerals that read the same from either end are much more common. Among those written with three digits, there is at least one in every decade, as illustrated by 121, 131; 343, 353; 585, 595. Two interesting and similar-looking palindromic numbers are 101 and 1001. The first one is prime, while the second one is not only composite, but also has as its prime factors the consecutive primes 7, 11 and 13.

Although palindromic numbers are common, it is not as well known that we can often reach a palindromic sum by adding a number to its reversal (the number obtained by reversing the digits) and continuing the process until the palindrome is achieved. The following is an example:

$$
\begin{aligned}
257 + 752 &= 1009\\
1009 + 9001 &= 10010\\
10010 + 01001 &= 11011
\end{aligned}
$$

Thus, the number 257, when added to the number obtained by reversing the digits and continuing the process, reaches a palindrome in three steps. On the other hand, had we selected 275 as an example, the process would have required five steps:

$$
\begin{aligned}
275 + 572 &= 847\\
847 + 748 &= 1595\\
1595 + 5951 &= 7546\\
7546 + 6457 &= 14003\\
14003 + 30041 &= 44044
\end{aligned}
$$

The palindromic property of numbers is an interesting characteristic of numbers, and an enthusiastic teacher might easily encourage his pupils to move further into an analysis of this phenomenon. Probably the mathematically inclined teacher, after illustrating the property, would raise such questions as:

1. Will other numbers besides those used here produce the palindromic sum 11011 or 44044?
2. Are randomly selected palindromes, such as 43234 or 9669, the palindromic sum of a number?
3. Can you tell by inspection whether a number becomes palindromic after relatively few steps?

Let us briefly examine these questions.

An investigation of palindromes reveals that at least nine separate three-digit numbers, when treated as described above, will produce the palindrome 11011 in three steps. Some of these are: 158, 257, 356, and 455. The obvious common property is that the middle digit is 5, and the one's and hundred's digits total nine. Thus, one would expect 554 and 752 to act similarly. It turns out, of course, that they do, as do 653, 851, and 950.

Another set of three-digit numbers that will produce a palindrome in three steps is {168, 267, 366, 465.....} . The common properties again are abvious, and pupils could be expected to investigate and find answers to these questions:

1. What properties are common among this set?

2. What is the palindromic total?

3. If 168 will produce a palindrome in three steps, then what can you conclude about 861?

The number 174, when it is added to its reversal, and the process is continued, will produce a palindrome in four steps. Other numbers that act similarly are 273 and 372.

1. Can you find the two other numbers that act similarly? According to the pattern, 471 and 570 would be expected to fit into this group, and they do.

2. Why would there be only five of these rather than nine, as in the sets we have looked at previously?

The latter is a good question, and the type that many pupils might see but have difficulty in putting into words. Actually, in this case, the first and last digits total five, and there are only five pairs of numbers that give this total. When the first and last digits total nine, there are nine such pairs.

This initial analysis may be sufficient for a given class, but what if it sparks real interest for a whole class, or even a small group in the class? Further investigation is certainly possible, and it can be a gruitful discovery experience. The process provides extensive drill in addition for classes that need it. On the other hand, it can also become a significant exercise in the use of the adding machine, if this is part of the teacher's scheme.

The most useful further investigation might be a follow-up of questions from the members of the class. However, if nothing specific arises, here are some questions that could be explored by a class, by separate groups, or even by separate individuals. One could expect such an investigation to uncover a variety of relationships in addition to those suggested here.

1. Given a palindrome such as 454, can you find a number that will produce it?

The answer here will probably be no. It is not known, at least to this author, if all palindromes can be generated by the process illustrated here. In the case of 454, considerable experimentation would be necessary to form a conclusion. On the other hand, 464 is obviously a palindrome formed from 331 in one step.

2. Can you categorize two-digit or three-digit numbers according to the number of steps required to reach a palindrome?

Considering two-digit numbers, it can readily be seen that if the digits add to nine or less, then a two-digit palindrome is reached in one step. Example: 23 + 32 = 55 or 43 + 34 = 77. Collecting the data in a table similar to the one in table 1 will help pupils draw conclusions and report their results. Two-digit numbers with the digits totalling a number greater than sixteen are special cases. There are only two numbers in the category whose digits total seventeen--these are 89 and its reversal 98. They do not produce a palindrome until the 24th step. The resulting number is 8,813,200,023,188.

TABLE 1
TWO-DIGIT NUMBERS

Total sum of the two digits	Examples	Number of steps to become palindrome
3 to 9	12, 42, 54	1
10	46, 37, 28	2
11	56, 47, 83	1
12	48, 57, 93	2
13	?	?
14	?	?
15	?	?
16	97, 79	6

The only number whose digits total 18 is 99 and it is already a palindrome.

3. Can you find a palindrome that cannot be produced by the process of adding a number to its reversal?

Perhaps the most obvious answer to this is the number 131. By experimenting with pairs of two-digit numbers that total 131, it can be seen that no pair will result in a number added to its reversal. The ones digit of 1 should give a clue. For example, 4 + 7 gives a ones digit of 1. Therefore, the number 47 added to its reversal 74 might be a possibility. However, the total 74 + 47 = 121.

It has not yet been established that all numbers treated as described earlier produce palindromes. Gardner states that the number 196 was carried to 4,147 steps with the aid of a computer and still did not produce a palindrome (Martin Gardner, "Mathematical Games." Scientific American (August 1970):110-13).

4. Can you find a palindrome that is also a prime? A square?

It is conjectured that there is an infinite number of palindromic primes. Examples of primes are: 101, 181 and 30103. For squares, 11^2 is palindromic, as is 22^2.

The development of a "friendliness toward numbers" that dispels fear of mathematics and mental blocks often comes with exposure to games and recreations. Perhaps the palindrome property may provide an avenue for analyzing numbers and discovering relationships that will set some children on the road to acquiring an intimacy that will be lifelong in nature--a permanent friendship.

TEACHING TOOLS:
HOW TO USE THEM IN MATH*

Here are some tips on ways you can generate excitement in your mathematics class.....by using teaching tools creatively.

It would be a mistake to say that teachers aren't using teaching tools in math. They are. What they're not doing for the most part is using these tools creatively.

I'm not talking about those teaching tools specifically designed for mathematics. There are, I'm happy to say, a growing number of teachers who are creating excitement in their math classes with such teaching aids as the number line and Cuisenaire rods. Fine. The more excitement we can generate in our math classes, the more mathematics our children will learn.

What I'm referring to are the familiar teaching tools like the overhead projector, the tape recorder and the teaching machine. They practically cry out for creative handling..... but when it comes to teaching mathematics, the cries all too often go unheard.

Take the overhead projector, for example. A teacher will sit down next to it and begin writing as she explains this or that mathematical operation. The children look at the screen and every so often the teacher looks up over her light at the children. The teacher's using the overhead projector to be sure, but as a substitute for the chalkboard, not as an avenue to creative teaching.

Certainly, the textbook will remain your primary teaching tool. But excitement is the name of the game, whether you're teaching mathematics or language arts or social studies. The sophisticated teaching tools available to us today can generate excitement, provided they're used creatively and not as a substitute for something else.

How can these tools be used creatively? Well, let's take a look at a few examples.

Overhead projectors:

First, ther's the overhead projector. Its uses are many: Diagrams and models can be prepared ahead of time and shown at the opportune moment during the lesson. Multiplication facts can be graphically illustrated by drawing a 9 x 9 rectangular array and uncovering only those facts you want to demonstrate at a particular time; for example, $3 \times 4 = 12$ or $5 \times 3 = 15$. By making transparencies such as these, you'll be able to give the children daily multiplication drill with a minimum of effort.

In the primary grades, you can place on the projector three-dimensional objects such as jacks to be counted and placed in sets. While the activity is being projected at the front of the room, the children can follow the procedures with counters at their own desks.

Here's another activity you can develop with the overhead projector: Draw two concentric circles with such numbers as 64, 144, 16, 40, 48 and 72 written in the inner circle. Then, each day make up a different rule for your students to follow in filling in the outer circle. For example, if the rule were $n + 4$, the student would have to write in 68 in the appropriate section of the outer circle; if the rule were $n \div 2$, the correct answer would be 32.

Teaching Tools:
How To Use Them In Math*

There's one great advantage to using the overhead projector for reinforcement. Since you're facing the class, you're in a position for an on-the-spot evaluation of your youngsters' progress.

There are many transparencies produced commercially for mathematics study. But it's really just as easy to make your own. Just try a few and see what I mean.

Tapes and film loops:

Tape recorders are an excellent means of reinforcement, especially when you're dealing with the auditory learner, the child who usually becomes bogged down with a lot of reading.

By programing the tapes— that is, by allowing intervals between the questions and answers — you can provide your youngsters with wonderful drill sessions on all of the basics of mathematics. It works this way: The student hears the question asked on tape and writes down his answer: then, after a suitable pause, the tape gives the correct answer, enabling the student to correct his answer. Since the student can do the drilling all by himself, there need never be the embarrassment of giving the wrong answer in front of the class.

As is the case with overhead projector transparencies, there are plenty of tapes available commercially — and plenty of opportunities to make your own.

For single-concept teaching, 8-mm film loops can be used to maximum advantage. These colorful, animated loops provide mental pictures that are hard to forget. When film loops are used in a class presentation, the teacher can stop the action, ask questions and then continue when she's satisfied the children have grasped the point.

Film loops are even more effective when worksheets are prepared for use by the individual learner. You'll find that loops are especially good for students who have difficulty arriving at generalizations after seeing specific examples.

Teaching machines:

Complete programmed visual and audio presentations can be provided by a teaching maching. This machine will never replace you, of course—but it will make life a lot easier by enabling you to reach those students who just aren't "turned on" by chalkboard and textbook.

Many materials prepared for the teaching machine have built-in methods of diagnosing where a student needs to start and what kind of instruction and practice he needs to learn a skill or concept. The machine also provides tests to determine just what the student has learned.

Teaching machines accomplish a great deal more than just organized drill. They show ways of relaing facts to other concepts and to families of facts. And since a correct response is immediately recognized, the machines provide instant reinforcement.

One good thing about a teaching machine is that the student is never intimidated. Even if he gives the wrong answer, the machine won't get angry or change its tone of voice. It will just repeat the same message over and over again until the right answer is given. How many teachers can claim the same measure of self-control?

Teaching Tools:
How To Use Them In Math*

Demonstration materials:

The more concrete the materials you bring into class, the more creative your teaching will be. For example, the "Count-A-Ladder," (EduKaid of Ridgewood) can be used to show sets of objects and number of sets. It's also useful in demonstrating the facts of addition, subtraction, multiplication and division.

An abacus can be used to teach place value. Youngsters can see, first hand, ten 1's being traded for one 10, or ten 10's being traded for one 100. And renaming in subtraction can be shown on the abacus where one 10 is changed to ten 1's, and so on.

The same materials used in class demonstrations can be used by some learners in doing their daily lessons. In so doing, they'll find these aids will help them develop a method of working out these problems.

What it all adds up to is that the role of the teacher is changing.... from instructor to guide. No longer can she stand in front of the class and just teach. Today she's expected to diagnose, prescribe and evaluate.

In order to handle these expanded functions, the teacher needs all the help she can get from the equipment and materials developed for education. Students will no longer be "turned off" by mathematics if the teacher can create action in the classroom. And making use of the materials and equipment at our disposal is surely the way to do it.

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* From: Grade Teacher, Feb. 1970; pp 126 - 130, By Lola J. May

COULD AMANDA BE A TURTLE?
And Other Mathematical Mysteries*

The trouble with all too many enrichment activities in mathematics is that they're nothing but drill. Drill has its place, of course. But enrichment activities should do more than provide students with opportunities to repeat and repeat until they finally have a computation down cold. Enrichment activities should help develop thinking skills, problem-solving ability and opportunities for the child to express himself either on paper or verbally. I try to provide such opportunities in my mathematics enrichment program. The means I've chosen: Simple little stories from which rather sophisticated thinking problems can be drawn. Some of the ones I like best follow. They can be duplicated and handed out to children with some time to fill or used with the entire class as oral exercise or written assignment.

Could Amanda be a Turtle?

Antoinette is six years old. Adeline is three years older than Abigail. Abigail is four years old. Althea is three years younger than Antoinette. Amanda is five years younger than Adeline.

Questions:

1. Who is youngest? (Amanda)
2. How many different names are in the story? (Five)
3. How old is the oldest? (Seven)
4. Antoinette is how much older than Amanda? (Four years)
5. How old is Althea? (Three)
6. How old was Amanda five years ago? (She wasn't born yet)
7. Considering the story, is it possible that Amanda is a turtle?

Sharp Sharks

Six sharks ate a total of 14 fish. Van Etten ate three. Virgil ate one less than Van Etten ate. Victor ate one less than Virgil ate. Vincent ate two more than Virgil ate. Video ate two less than Van Etten ate.

Questions:

1. How many fish were left for Vesuvius to eat? (Three)
2. Who ate the most fish? (Vincent)
3. If Virgil had eaten one fish less than he did, then how many of the sharks would have eaten only one fish? (Three)
4. After all the sharks finished feeding, which one was probably the least hungry? (Vincent
5. If the first shark ate one fish and each subsequent shark ate twice as many fish as the one before him, how many fish would have been eaten altogether? (63)
6. Mr. Jones had shark steak for dinner. Van Etten, Virgil, Victor, Vidio and Vesuvius are still swimming around. Who is missing? (Vincent)
7. If Vincent ate Virgil, then Vincent would have eaten how many more fish than did Vesuvius? (Four - don't forget Virgil)

Ach du lieber Octupus!

There are three octopuses. If you add their ages together, you will get 12. The octopuses are names Tobiah, Theodore and Thaddeus. Tobiah is three years old. Theodore is two years older than Tobiah.

*From: Grade Teacher, February 1970, pp. 61-64, by Ann C. Stone.

Questions:

1. Which octopuses are the same age? (None)
2. Is Thaddeus older than Tobiah? (Yes)
3. Thaddeus is how many years older than Theodore? (He is one year younger)
4. Is the octopus with the shortest name the youngest? (Yes)
5. Tobiah, Theodore and Thaddeus have how many arms altogether?(24)
6. Tobiah, Theodore and Thaddeus have 12 children. Tobiah has one more child than Thaddeus. Theodore has two children more than Thaddeus and one more than Tobiah. How many children do Tobiah, Theodore and Thaddeus each have? (Tobiah - four; Theodore - five; Thaddeus - three)
7. If Tobiah, Theodore, Thaddeus and their children all locked arms, how many armlocks would there be? (120)

Counting Crocodiles

Nathan is not as old as Nigel. Nigel is not as old as Noah. Noah is 10 years old. Nathan, Nigel and Noah are all crocodiles. The youngest crocodile is five years younger than the oldest. Nigel's age is equal to that of Nathan subtracted from that of Noah, plus one.

Questions:

1. How many years have the three crocodiles lived? (21)
2. How old will the youngest crocodile be in three more years? (Eight)
3. What is the name of the oldest crocodile? (Noah)
4. How old is Nigel? (Six)
5. How much younger will Nigel be than Noah when Nigel is 12 years old? (Four)
6. If Noah lives to be 50 years old, what will the combined ages of Nathan and Nigel be 10 years after Noah's death? (101)

Snakes Alive!

Horatio has three zebras more than Hannibal has and two less than Horace. Hannibal has four zebras. Horatio, Hannibal and Horace are snakes.

Questions:

1. If each snake in the story loses one zebra, then how many zebras will there be? (17)
2. Who owns the most zebras? (Horace)
3. Horace has how many more zebras than Hannibal? (Five)
4. Why can't Hannibal give six zebras to Horatio? (He only has four)
5. If Horace gives all his zebras to Hannibal, then how many zebras will Hannibal have? (13)
6. If Horatio were twice as long as Hannibal and Horace was one-third shorter than Horatio, which snake would be the shortest?
7. Why do you think the snakes keep zebras? (?)

Who's Pulling My Leg?

Genevieve has two legs less than Georgianna. Gwendolyn has three legs more than Genevieve. Georgianna has nine legs.

Questions:

1. What is the total number of legs involved in the story? (26)
2. What might Genevieve, Georgianna and Gwendolyn be? (Insects)

3. Who has the fewest legs? (Genevieve)
4. How many legs would Gwendolyn have if she had only half her present number? (Five)
5. If Genevieve gets another leg, how many will she have? (Eight)
6. Does Genevieve have more legs or fewer legs than Gwendolyn? (Fewer)
7. How many legs would Gwendolyn have if she had double her present number? (20)

Pop Go the 'Sicles!

Four girls have 12 Popsicles between them. Ophelia has two more than Octavia. Octavia has none. Olive has three more than Ophelia. Olga is the name of the fourth girl.

Questions:

1. How many Popsicles does Olga have? (Five)
2. Which two girls own the same number of Popsicles? (Olive and Olga)
3. What do Ophelia, Olive and Olga probably like to do? (Eat Popsicles)
4. If Octavia gets two Popsicles from each of the other girls, how many will she have? (Six)
5. Octavia doesn't have any Popsicles but doesn't seem to care. How might this be? (She has already eaten several Popsicles)
6. Octavia and Olive are the same age. Ophelia is one year younger than Octavia and Olive. Olga is one year older. Their total age is 28. How old is Ophelia? (Six)

Prehistoric Puzzler

Ferdinand has a green dinosaur for a pet. Frederick has a smaller blue one. Franklin has a red dinosaur that is larger than Ferdinand's.

Questions:

1. In what way are Ferdinand, Frederick and Franklin unusual? (They have pet dinosaurs)
2. Who has the largest dinosaur? (Franklin)
3. Is the red dinosaur smaller than the green one? (No)
4. What color is the smallest dinosaur? (Blue)
5. Name three colors that are not mentioned in the story. (Purple, brown, orange)
6. If Ferdinand gets a new yellow dinosaur, there will then be a total of how many dinosaurs in the story? (Four--unless Ferdinand, Frederick and Franklin are dinosaurs, too; then there will be seven)

Which Witch?

Wilhelmina, Winifred and Wisteria are all witches. Winifred keeps a bat as a pet. Wilhelmina has a broomstick. Wisteria has a black cape and is 200 years old. The witch who can fly is 200 years older than Wisteria. The witch who owns a pet is half as old as is Wilhelmina.

Questions:

1. What is the combined age of the witches? (800)
2. Which witch is oldest? (Wilhelmina)
3. What does Wisteria own? (A black cape)
4. How many years will it be before Wilhelmina is 600 years old? (200)
5. How old will Winifred be in the year 2000? (230)
6. If Wilhelmina ages only half as fast as Winifred, and Wisteria ages twice as fast, what will be the combined ages of the witches in the year 2000? (905)

Bridging the Gap

Esmeralda has nine gold teeth. She sells three of them. Later on, she buys two more. Now she has five more teeth than Ermengarde.

Questions:

1. How many teeth does Ermengarde have? (Three)
2. How many teeth does Esmeralda end up owning? (Eight)
3. What color are Ermengarde's teeth(?)
4. Who is "she" in the story? (Esmeralda)
5. What does Esmeralda get in exchange for her teeth? (Money)
6. Why might it be difficult for Ermengarde to eat steak? (She has only three teeth)

BOTTLE CAPS GO TO SCHOOL*

It all began with show and tell . . . but ended up as modern math.

Clink! Clinkey-clank! Metal bottle caps spilled out on our table tops. Eager voices filled the kindergarten room. This was the day the five-year olds had been awaiting so impatiently--the day when we would play the "Bottle Cap Game."

It all started when Billy brought a small collection of bottle caps for "Show and Tell." As he showed them to the class, we noted their colors and brand names. We counted them one by one into Billy's hand.

I said: "Do you suppose we could make a large collection of bottle caps? If so, I know a special game we can play." And that was the beginning of an activity that led to many worthwhile number and perceptual experiences.

Daily the bottle caps arrived. Each child's contribution meant a new counting experience--sometimes by ones, sometimes by twos and sometimes even by fives.

The children were encouraged to think cognitively. They categorized and compared--bottle caps were classified by color and by brand name. The children made quantitative judgments--they decided whether one group contained more than, less than or the same as another group. Were there more blues than reds? Fewer 7-Ups than Pepsi Colas?

Number terms such as few, fewer, fewest, great, greater, greatest, less, more, many, large, larger, largest, small, smaller, smallest, equal, the same, unequal had real significance during our daily "collection" time.

Simple computation came naturally: two root beer caps plus one orange soda cap made three--very visually, concretely and meaningfully. The children clearly saw that Ruth needed two more caps to have the same number as Ronald or that Patty had one less than Tommy. The children added and subtracted, arranged items into sets, separated, put sets together, grouped and regrouped the bottle caps. They were dealing with basic number principles and having fun doing it.

Each day the question was asked: "Do we have enough caps to play the game?" One day they heard the answer they wanted--yes!--and out came our shoe box filled with caps.

The children were divided into six teams (that was the number of tables in the room). I tried to make sure each team was equitable as far as ability was concerned. I labeled each table with a large cardboard numeral. Each child was to select from a pile and place them in front of him. All extra caps were returned to the shoe box.

Then the game began. The children were instructed to push their caps together in a pile. "Now watch," I said. "I am going to write a numeral on the chalkboard. Then you will take that number of bottle caps and put them in a line on the table in front of you. You must know the numeral, of course, because it tells you how many caps you must use." By this time, the children had had several months' experience in learning and using numerals. The game would put this experience to the test.

Next I explained that as soon as a child had completed the operation he was to stand up by his seat, to show that he was finished.

Most important of all, I stressed that this was a team effort, that the team that finished first would win, not the individual. Therefore the children were encouraged to help each other.

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* From: Grade Teacher, January 1970, pp. 2-22, by Enid P. Bloome.

This was a difficult concept to get across. The children had a tendency to jump up and scream: "I got it. I'm finished first!" I explained that I did not want to hear "I'm finished, but instead, "We're finished."

Enthusiasm and excitement ran high. We practiced the game several times, just to make sure everyone understood the directions--and then competition started. It was gratifying to see the more able team members helping the less capable, and the slower ones accepting the help with no embarrassment. It was great to see the children developing team spirit and cooperating with each other, rather than competing.

At the end of the game points were counted and the winning team was applauded. Win or lose the children loved the game.

We developed many variations on our basic bottle cap game. For example, we built the game around addition, subtraction, equal and unequal groups, seeing numbers as groups of twos, fives and also as 10-plus (10 and 1, 10 and 2, etc.). There's a good deal of variety in simply letting the children take turns being the "teacher," too.

The bottle caps lend themselves to copying patterns and shapes. One day I put a design on the board and asked the children to copy the same pattern with their caps. I started with a simple design--two rows of five each. Then I made another design of three rows of 3 and one row of 1. My next design was a pyramid.

This was more difficult. It was interesting to see many children come to the board, touch the "caps" I had drawn and count the number in each row. I knew then that they felt relaxed and free and under no rigid pressure. The atmosphere was one of give-and-take as children looked to each other for help.

The shapes became increasingly more difficult. We tried a square, a triangle and a criss-cross. Next there were letter shapes: L. O. N. H and so on.

Many areas of perceptual skills and visual discrimination were being brought into play, such as eye-hand coordination, spatial relationship, direction and reproduction of shapes. But, once again, the learning was taking place in an excited but happy and relaxed, fun-time atmosphere.

Come to think of it, who wouldn't have fun after drinking all that soda pop?

VARIATIONS ON MATHEMATICAL THEMES*

All teachers are faced with this question: "What will I do today for practice that is different?"

Trying to vary the practice period is difficult even for the most creative teacher. Most of the time all you need do is adapt some of the familiar types of practice. As you read some of the suggestions in this article, they will seem so simple you will wonder why you did not think of them. The answer is: "You can, once you see how simple variations can be made."

One way of practicing the many names for a number is to write a number on the chalkboard and ask the students to give it other names. As the students call out other names the teacher records them on the chalkboard. Sometimes the students are given a sheet of paper with the number to be named in many ways written in the middle. Then they write all the names they can (Fig. 1).

A variation to this activity is to make up a paper with the number to be named in many ways but start the names and force the pupils to all find the same names (see Fig. 2). This is more difficult, but it requires them to think— and, they are practicing many facts. In this way the teacher can control the facts to be practiced. In the middle grades the same activity can be used by making the number in the square a larger number and requiring more multiplication and division as well as subtraction and addition (see Fig. 3).

Another variation to practice is to give the students sets of phrases. They must match the ones that name the same number (see Fig. 4,). The phrase 6 + 1 is matched to 10 - 3 because they both name the same number, 7. The phrase 4 x 2 is matched to 56 divided by 7 because they both name the number, 8.

Puzzling the students

Cross number puzzles appeal to many students. The variation of this is to set up an activity where the pupils fill in numbers to make both the horizontal and vertical equations correct. In the puzzle shown (Fig. 5) the students fill in numbers in the empty squares to make all the equations true. In the top row they might use 2 + 2 + 2 = 6. The next row could be 3 + 1 + 3 = 7 and the last row 1 + 4 + 4 = 9. Whatever the order of numbers used, they must check to see if the vertical equations are also true. If they have chosen the wrong numbers they will have to try other combinations. This type of problem requires trial and error as well as the ability to compute. Fig. 6 shows a similar puzzle, while in Fig. 7 the students have to fill in the operation symbols + or - to make the sentences true, both horizontally and vertically.

Make up Puzzles:

Puzzles should be made up that require multiplication and division. Some of the brighter students should be asked to make up puzzles for the other students.

Problems that take two operations are a real challenge. Frame arithmetic is used a great deal in elementary mathematics but sometimes letters can be used in place of the frames to stand for numbers. The letters x and y stand for two numbers. The phrase, x + y, means you add the numbers and x - y means you subtract the two numbers.

Variations On Mathematical Themes*

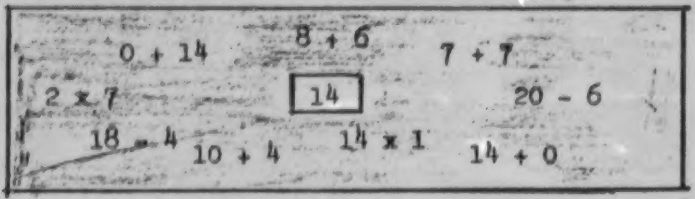
0 + 14
8 + 6
7 + 7
2 x 7
14
20 - 6
18 - 4
10 + 4
14 x 1
14 + 0

Figure 1

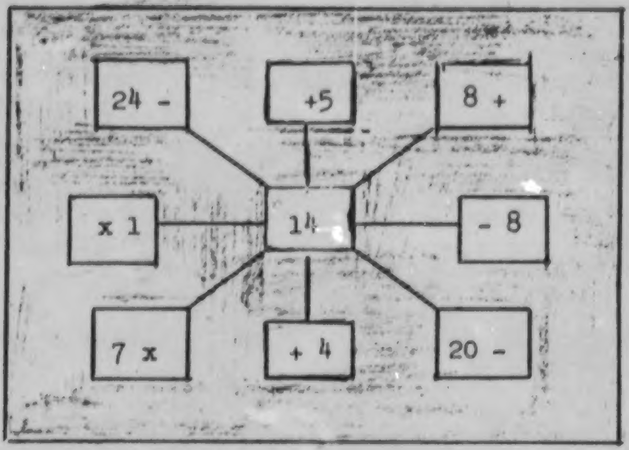
24 -
+5
8 +
x 1
14
- 8
7 x
+ 4
20 -

Figure 2

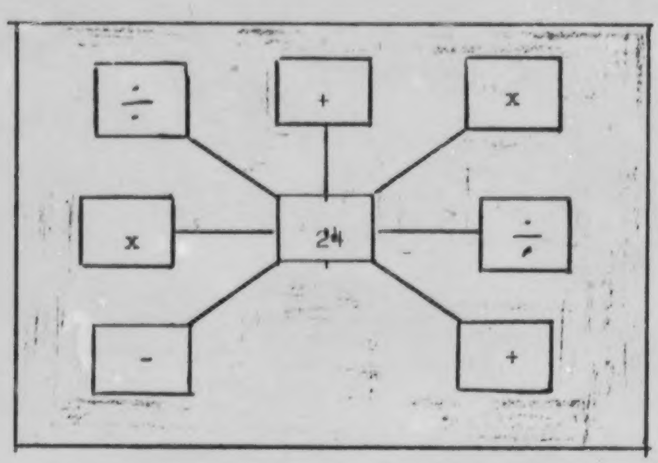

Figure 3

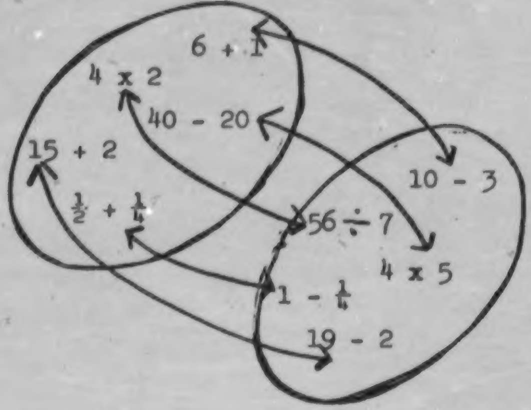
6 + 1
4 x 2
40 - 20
15 + 2
$\frac{1}{2} + \frac{1}{4}$
10 - 3
56 ÷ 7
4 x 5
$1 - \frac{1}{4}$
19 - 2

Figure 4

2	+		+		=6
+		+		+	
	+		+		=7
+		+		+	
	+		+		=9
=6		=7		=9	

Figure 5

4	-		+		=6
-		+		-	
	+		-		=4
+		-		+	
	+		-		=5
=5		=3		=7	

Figure 6

Variations On Mathematical Themes*

2	▨	6	▨	7	=1
4	▨	2	▨	5	=1
5	▨	4	▨	7	=8
=1		=8		=9	

Figure 7

In Fig. 8 students have to think what two numbers when added make 10, but when you subtract the remainder is 2. The answer is 6 and 4. In the next problem you have to find the two numbers whose sum is 8 and difference is 4. This time the answer is 6 and 2. In the last problem you have to find two numbers whose sum is 9 and difference is 5. The two numbers are 7 and 2. Fig. 9 shows how to use the tables with two operations. In these, the students have to really think and also be careful readers.

x	y	x + y	x - y
		10	2
		8	4
		9	5

Figure 8

x	y	x + y	x times y
8		14	
		17	72
	9		54

Figure 9

x	x + 1	1 + x	x - 3	9 - x	x + 2	17 - x
5						
8						

Figure 10

One number, many answers:

Using the letter x to stand for some numbers you can make a table where students have to do many problems with the same number (Fig. 10). To fill in the right numbers in the table, the pupils have to be careful readers, for 9 - x is not the same as x - 9. It is good to have activities that require students to look carefully at the operation signs and think before they find the answers. Other tables can be made up where they have both x and y in the table (Fig. 11). Now they must be careful which numbers they are adding and subtracting. Attention is drawn to the idea that x + y is the same as y + x. This reinforces the commutative property of addition in the general form. The activity also shows that y - x is not the same as x - y unless x and y stand for the same number.

In learning the order of numbers students are taught the symbols for inequality. The symbol "<" means "is less than" and the symbol ">" means "is greater than." Activities can be made up where a variable is between two inequality symbols. A set of numbers is given and the students must decide which of these numbers satisfies the sentence. The sentence in Fig. 12 says x is larger than 10 and less than 40. Looking at the set of numbers, 17, 20 and 30 can be chosen to make the sentence true because they are less than 40 and greater than 10. The number 10 in the set does not make the sentence true because 10 is not greater than 10. This may seem difficult the first time but soon the children see the conditions must both be true if the number is going to satisfy the whole sentence.

x	y	x + y	x - y	y + x	y - x	x + y + 4
9	5					
15	2					
13	13					
16	0					

Figure 11

Variations On Mathematical Themes*

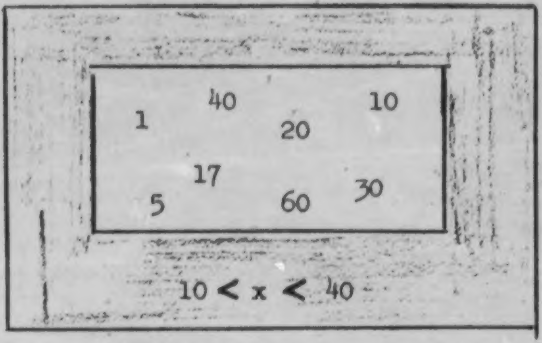

Figure 12

Making tables that require more than one operation, changing squares and triangles to x and y, changing the cross number puzzles to include equations are all small variations but they will add new life to the practice period. Try these— then be brave and make up some of your own.

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* From: Grade Teacher, Feb. 1969, Dr. Lola May, pp 127-130.

WHAT'S IN A NAME?*

Plenty! You can do away with arithmetic drill work and replace it with a fun game for your youngsters.

All too often, students are given arithmetic drill work which is just plain tedium--an ordeal of adding column after column of figures with little, if any, immediate payoff.

Why subject your youngsters to that when you can make it fun by playing "Name Arithmetic"?

Few things are more familiar to a child than his name. So, to get your students' immediate interest, start with that.

On the chalkboard assign a value to each letter of the alphabet (See figure 1). Ask each youngster to print his first and last names on a sheet of paper (no nicknames, please). Then ask each student to substitute the appropriate numbers for each letter in his name (see Figure 2) The next step is to add these number values to give each student the full numerical value of his name. For example, in Figure 2 the value of MARY CARPENTER is 157.

Use this introduction as the basis for the followup questions and activities:

A	B	C	D	E	F	G	H	I
1	2	3	4	5	6	7	8	9
J	K	L	M	N	O	P	Q	R
10	11	12	13	14	15	16	17	18
S	T	U	V	W	X	Y	Z	
19	20	21	22	23	24	25	26	

Figure 1

M	A	R	Y	C	A	R	P	E	N	T	E	R
13	1	18	25	3	1	18	16	5	14	20	5	18

Total: 157

Figure 2

1. Whose name gave the greatest value in the class? Whose name gave the least value in the class?
2. How many students have odd values for the sum of their coded letters? How many students have even values for the sum of their coded letters? Are these two results reasonably close? Is this to be expected?
3. How many students have odd values for both first and last names?
4. How many students have even values for both names?
5. How many students have an odd value for one of their names and an even value for the othe
6. How does the total number of students in Question 5 compare with that in Questions 3 & 4?

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* From: Grade Teacher, By Ernest R. Ranucci, pp. 48-50.

7. In how many cases does the value of the given name exceed that of the family name? In how many cases does the value of the family name exceed that of the given name? Which is more likely to occur? Does national origin have anything to do with this? Explain.

8. Reexamine the numerical values of the letters in your name. Write down an "O" for those which are even (Mary Carpenter; 13-1-18-25-3-1-18-16-5-14-20-5-18 becomes OOEOOOEEOEEOE). Now proceed from left to right (that is, from letter to letter). Give yourself one point for every change in status--from odd to even (O to E) and from even to odd (E to O) count one point each; from even to even (E to E) and from odd to odd (O to O) count nothing. Accordingly, the value assigned to Mary Carpenter's name is 7. Who has the highest total in this game? Who has the lowest total? Is there anyone whose variations total zero? If not, could such a thing happen? (All numbers are even or all numbers are odd.)

9. Have each child select a word with an agreed-upon number of letters, perhaps five. The contest now is to see whose word has the highest coded total, the lowest coded total. Possible variations might include five-letter words with only odd numbers, only even numbers, only even numbers, etc.

This game can lead to all kinds of interesting possibilities and complications. Encourage creativity. For example, if a youngster comes up with baa-ed (making a five letter word from the sound a sheep made), he's got a good low total of 13. If the student is that creative, accept it, by all means. The same goes for the youngster who comes up with wuzzy as in "Fuzzy-Wuzzy was a bear. . ." in a game to find the five-letter word with this highest value. Wuzzy totals 121, about as large a total as any youngster could hope for.

Note that many of these activities are only incidentally related to arithmetic, and in this lies their strength. Another advantage of these exercises is the fact that they involve fields other than mathematics--English and social studies, for example. The discovery and search for new words aids in vocabulary building. The investigation of family names may lead to fruitful learning. Activity 7 deals with the relative weights of family names and given names. Slavic names, for instance, are more likely to be heavy on the family name. Conversely, many Chinese family names are the two- or three syllable types--Ho, Li, Eng, etc., and thus likely to be lighter on the family name.

Questions 3, 4, 5 and 6 have an aspect of probability connected with them. With regard to the nature (odd or even) of the values of the given name and the family name, there are four possibilities:

First Name	Last Name
Odd	Odd
Odd	Even
Even	Odd
Even	Even

This means that, ordinarily, about half the class will have names with like status (odd-odd or even-even) while half the class will have names with unlike status.

Encourage your students to come up with other arithmetic games based on their own names.

Chess and Mathematics *

Chess is probably the oldest game in the world. For a thousand years and more, chess has captured man's imagination and intellect. Today it is played around the world, in some countries to such an extent that it might be called the national pastime. For a game to have lasted so long and to have been enjoyed by so many people of all ages and backgrounds, it must posses some very special characteristics.

Chess is a game of almost infinite variations. There are 197,299 ways of playing the first four moves in a chess game alone! But despite this vast number of possibilities, chess is controlled by rules which make it a very meaningful activity rather than a random exploration. Aside from the game aspect of chess involving competitive play and a pleasant pastime, much of the beauty that has captured man's imagination in the game is related to the mathematics underlying chess. In fact, mathematicians through the centuries have been fascinated by chess and written volumes of mathematical problems associated with the game. Among the famous men of mathematics who have concerned themselves with chess are Euler, DeMoire, Ball, Legendre, Pierce, Shannon, and Hardy.

The analogy between mathematics and chess is most clearly stated by W. W. Pierce Ball in "Mathematical Recreations and Essays":

"Modern mathematics is concerned with playing the game according to the rules; others may inquire into the 'truth' of mathematical propositions, provided they think they know what they mean.

The rules of the game are extremely simple. Once and for all the postulates are laid down. These include a statement of all the permissible moves of the 'elements'--or 'pieces.'

It is just like chess. The 'elements' in chess are the thirty-two chessmen. The postulates of chess are the statements of the moves a player can make and of what is to happen if certain other things happen. For example, a bishop can move along a diagonal; if one piece is moved to an occupied square, the other piece must be removed from the board; and so on. Only a very original philosopher would dream of asking whether a particular game of chess was 'true'. The sensible question would be, "Was the game played according to the rules?"

The moves of each of the pieces on a chessboard can be described mathematically in terms of vectors. Chess is used for studying vectors in Book 2 of School Mathematics Project, for example, just this way. The mathematics implicit in chess makes the learning and development of skills for playing the game a process closely akin to the learning of mathematics. Perhaps this is why so many mathematicians are chess players. It seems to follow, also, that playing chess would encourage mathematical thinking in youngsters.

G. H. Hardy, a famous British mathematician wrote in his book, "A Mathematical Apology":

"The fact is that there are few more 'popular' subjects than mathematics. Most people have some appreciation of mathematics, just as most people can enjoy a pleasant tune; and there are probably more people really interested in mathematics than in music. Appearances may suggest the contrary, but there are easy explanations. Music can be used to stimulate mass emotion, which mathematics can-

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* From: Cuisenaire Reporter, Spring 1972, pp. 10-12.

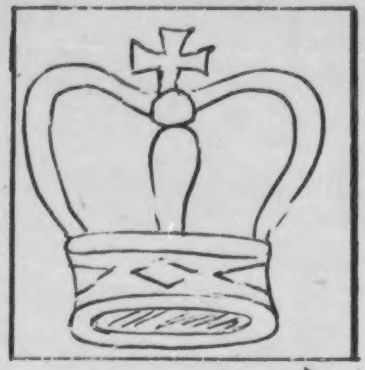
not; and musical incapacity is recognized (no doubt rightly) as midly discreditable, whereas most people are so frightened of the name of mathematics that they are ready, quite unaffectedly, to exaggerate their own mathematical stupidity.

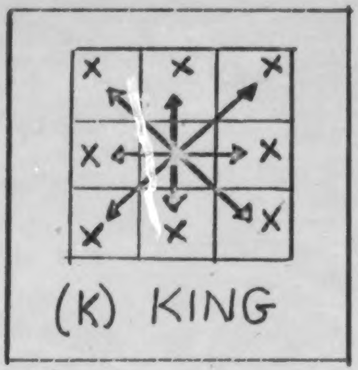
A very little reflection is enough to expose the absurdity of the 'literary superstition.' There are masses of chess-players in every civilized country--in Russia, almost the whole educated population; and every chess-player can recognize and appreciate a 'beautiful' game or problem. Yet a chess problem is simply an exercise in pure mathematics (a game not entirely, since psychology also plays a part), and everyone who calls a problem 'beautiful' is applauding mathematical beauty, even if it is beauty of a comparatively lowly kind. Chess problems are the hymn-tunes of mathematics."

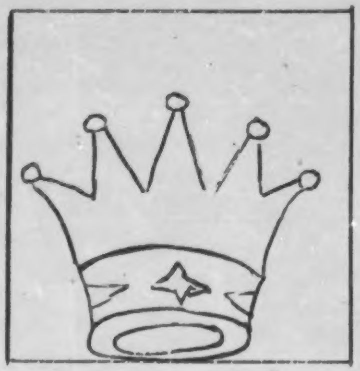
The principal reason chess is not played by far more adults and children has been the necessity to first memorize the moves of the game and then relate them to the traditional sculptured chess men. It's also necessary to remember how to place each chess piece on the board. These first steps have made the pleasure of playing chess and the sharpening of mental skills associated with it unavailable to many, particularly, to young children.

An engineer, Desmond Margetson, recently came up with a brilliant new set of chess pieces and chessboard which makes the game instantly available to anyone. These pieces and board are called QUICK MASTER^{TM} CHESS, and as diagrams here will show, they make the essential vector moves for each piece instantly visible. Unique color coding also makes the initial setup of the pieces on the playing board obvious. By means of this new set, school children can start playing chess instantly without first laboriously memorizing the rules. Thus the mathematics implicit in the chess moves become immediately explicit. Children as young as four have started playing chess with this new technique.

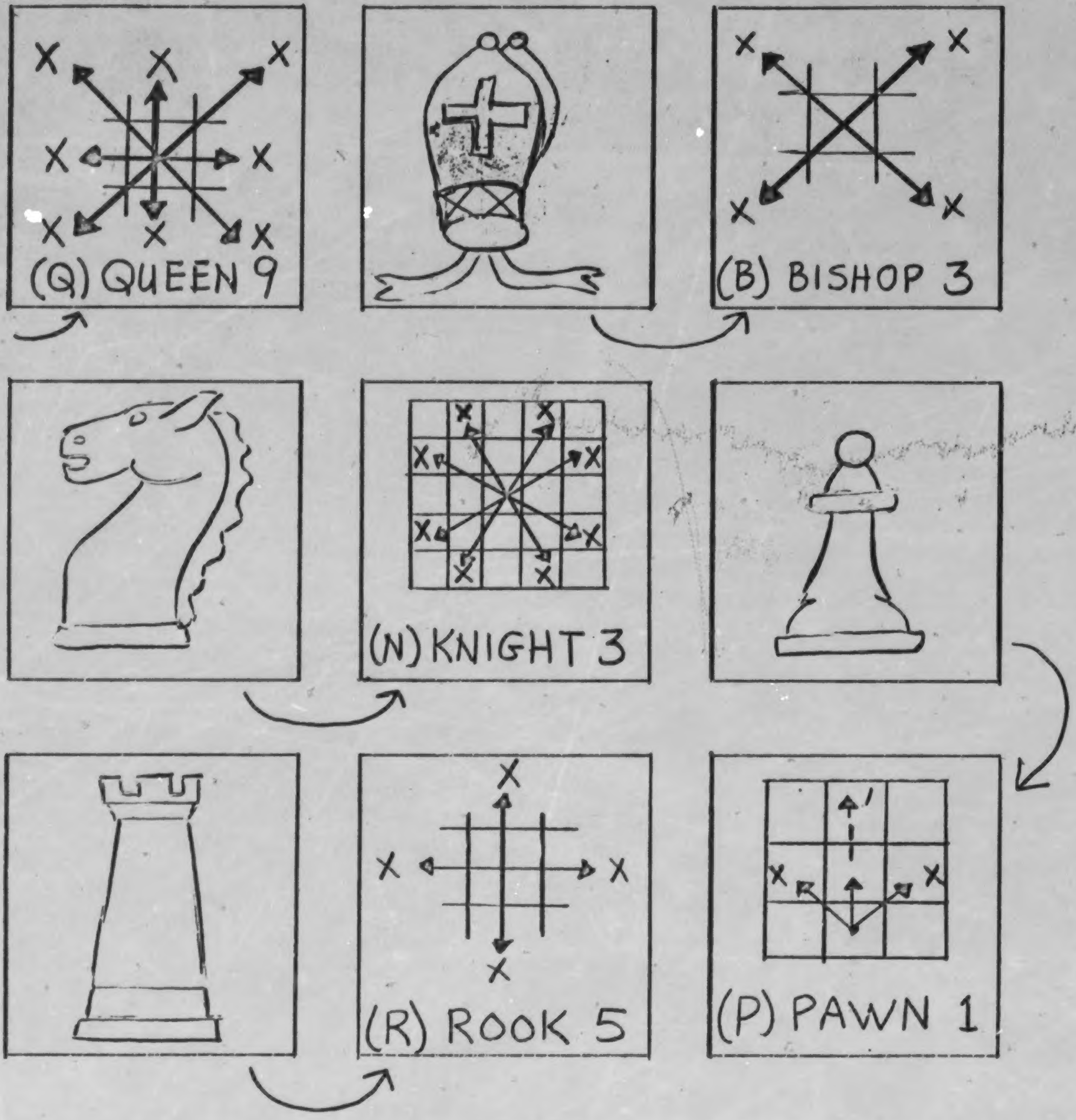
Now many elementary classrooms and math labs can set aside additional space for chess in their game corner.







Chess and Mathematics


(Q) QUEEN 9
(B) BISHOP 3
(N) KNIGHT 3
(P) PAWN 1
(R) ROOK 5

KINETIC ENERGY AND THE STORAGE OF DATA

Objects of different mass, falling from the same initial height above the ground, strike the ground with the same velocity but different kinetic energies. It is difficult to measure the velocity of falling objects because of experimental complications; it is relatively easy, however, to measure kinetic energy of falling objects and to compare these measured kinetic energies with a prediction derived from physical theory.

TASKS:

To gather quantitative data about the measured kinetic energy of falling objects
To study the data patterns.
To make predictions from the data.

PURPOSE:

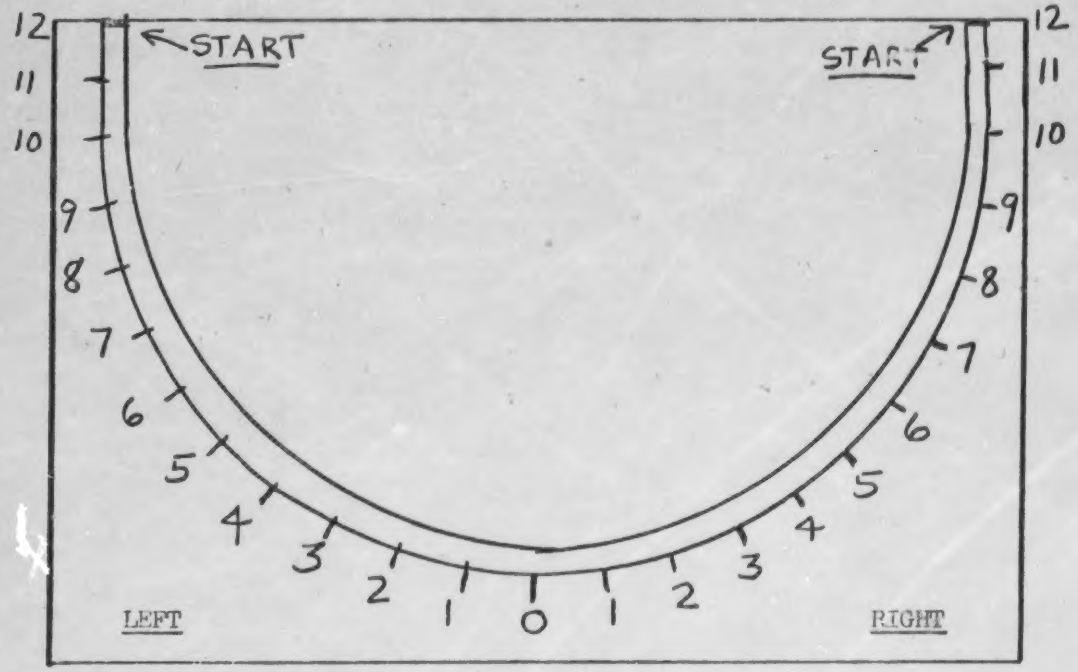
The mathematics involved in dealing with various physical phenomena is intriging and often presents discrepent data. Without attempting to explain the technical or scientific explanations (and complex formulas) for physical phenomenon data, you are asked to systematically gather data, record that data in one or more forms, and then look for patterns in the quantitative data in order to make predictions!

MATERIALS:

Lengths of "hot wheels" tracks.
Masking tape
Objects such as marbles, toy cars, balls, blocks, and any other objects which can be rolled or slid down the tracks.
Chalkboard or bulletin board.

PREPARATIONS:

1. Tape the "hot wheels" tracks against a chalkboard or bulletin board as shown in the sketch:



KINETIC ENERGY AND THE STORAGE OF DATA

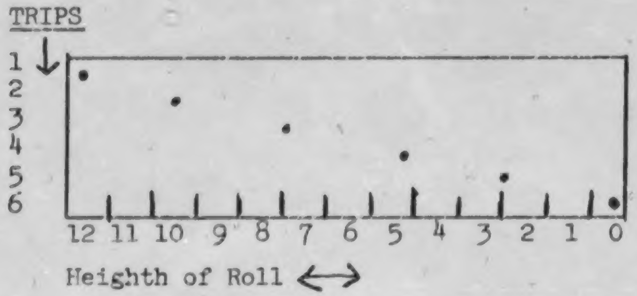
2. Below the track mark off a graduated scale as shown with zero (0) being the bottom-most point of the curve of the track -- this is the "resting place" of falling objects. Mark a scale to the RIGHT and LEFT of point zero up to 12 (or more). Mark a STARTING POSITION on both the Right and Left ends of the track at the point of the highest numeral on the scale.

3. Position yourself in front of and close to the apparatus so easy and fairly accurate observations can be made while you are recording data.

4. To prepare for recording data, roll a marble, starting at the LEFT starting position (release, but do not push the marble). Note how far the marble rolls "up the Right scale" (to what number), how far back to the left scale (to what number), and so forth until the marble comes to rest at position 0. (Note: It may not always come to rest at precisely 0 position.
A chart of your "TRIPS" may look like this:

TRIPS	FROM →		TO	
1. START:	LEFT	12	RIGHT	9
2.	"	8	"	7
3.	"	6	"	5
4.	"	4	"	3
5.	"	2	"	1
6. (stopped)	"	0		

(NOTE: Of course this is not an actual observed record of what happened, but if it were you would be able to note that the data are very symetrecal -- each trip to right and back to left shows a reduction in kinetic energy of the object. A graph of the above data might look like this:



These are only two ways you may wish to record observed data. Think of others. How might you record 3 successive sets of rolls with the same object? ...objects which appear to have some relationship, such as, a 1/2 inch marble, a 3/4 inch marble, and a 1" marble?

PROBLEMS TO TRY:

1. Roll a marble starting at LEFT 12.
   Record your data.
   Look for patterns in the data.
   Can you predict what the data might look like if you repeated the above roll but started at RIGHT 12?
   What did you observe?
   Record the data and compare with your first data.
   Was your prediction correct?

KINETIC ENERGY AND THE STORAGE OF DATA

(NOTE: If more than one observer recorded the data simultaneously -- without audible calling out of the data -- did you observe and record the same data?)

2. Roll two similar objects -- one at a time.
   Record the data for both rolls.
   What did you observe?
   Would you get the same data if you repeated the rolls?
   What can you predict?
   What happened? (Record your data and then summarize.)

3. Roll 2 objects of similar size but different weights (or other characteristics such as "no wheels", "not ball-like", "smooth", etc.
   Record your data.
   What did you observe?

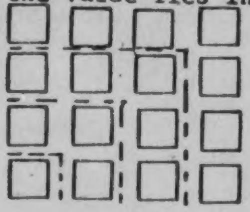
4. Roll 2 similar objects at the same time from the same starting point.
   Same starting point.
   Record your data.
   What did you observe?
   Could you predict that similar results will occur again?
   How about a 3rd roll?

5. What happens to the data when you "change the starting point" from LEFT 12 or RIGHT 12 to say LEFT 9 or RIGHT 7? Try some rolls, record your data, and compare it to previous data for that object when the starting point was Right 12 or Left 12. What did you find out? Any patterns in the data? Any predictions?

6. NOW create some of your own problems for data gathering. Roll and record data, look for patterns, make predictions, and re-gather and record your data to check predictions. You might try some of these:
   a. Vary size
   b. Vary shape
   c. Vary other characteristics (texture, etc.)
   d. Vary conditions
      How you release the object.
      Dryness or moistness of object. (Careful to not damage chalk board or other materials.)
   e. Vary starting points.
   f. Vary "wind" conditions
      (A fan or other means to create resistance.)

Search for Patterns Helps Children Think for Themselves*

One of the main tasks of a teacheris to present children with problems that will require them to analyze the part, ask questions, and then find patterns. This is exciting, and the value lies in letting children think for themselves.



1 = 1
1 + 3 = 4
1 + 3 + 5 = 9
1 + 3 + 5 + 7 = 16
1 + 3 + 5 + 7 + _ = __

The present the work shown above, draw one square on the chalkboard and then draw three more squares, to form a larger square of four small squares. The first question is, "How many squares would you have to draw to make the next largest square?"

As a child goes to the board to draw on the next five small squares, continue the' questioning until the drawing looks like the one above. Then record the work in equation form. The sentence 1 + 3 = 4 is a record of the first square, then the next three squares which form the larger square. After four sentences are on the board, as shown above, see if the children can write the next sentence by looking for the pattern. By asking questions, you should bring out in the discussion that the only numbers used on the left side of the equations are odd numbers and they are consecutive odd numbers. The numbers on the right side of the equations form a pattern of odd, even, odd, even, etc.

Some of the children should see why this pattern occurs: when add two odd numbers, the sum is even, but when you add three odd numbers, the sum is odd. Some child may generalize and say an odd number of odds always produces an odd number in addition and an even umbers of odds always produces an even number. Getting the children to look at the equations causes them to recall their knowledge of odd and even numbers.

The next pattern to lead them to is the number of addends in each sentence. In the second equation there are two addends and the sum is 4. In the third equation there are three addends and the sum is 9, etc. See if the children can find any patterns between the number of addends and the sum of each equation. The result of the discussion should be the discovery that 2x2=4, 3x3=9, and 4x4=16. When you have four consecutive odd numbers the sum is the same as 4x4 or 16. What is the sum of the first five odd numbers? Some child will show 25 immediately. Ask him to tell how he got the sum so fast. His explanation will be that 5x5 is 25. The brainwister is to ask the children to give the sum of the first ten odd numbers. Many should respond 100 because they see the pattern and know that 10x10 is 100.

List the numbers 1,4,9,16,25,36,49,64,81,100, etc., on the board and ask the children what name they would give to this set of numbers. ome will call it odd and even numbers because that is the pattern. Others will just make up funny names. But, if you are lucky, some child will say square numbers. This is the child who remembers that you used squares to make the numbers. This set of numbers is called square numbers.

Looking for patterns in this problem, children use their knowledge of idds and evens, they use addition, they use multiplication, and they learn about a new set of numbers in mathematics.

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*From: Grade Teacher, September 1967, pp. 113-114; By Lola May.

If the children become curious why odd numbers were chosen instead of other numbers, you can present another problem to show patterns which develop with other numbers.

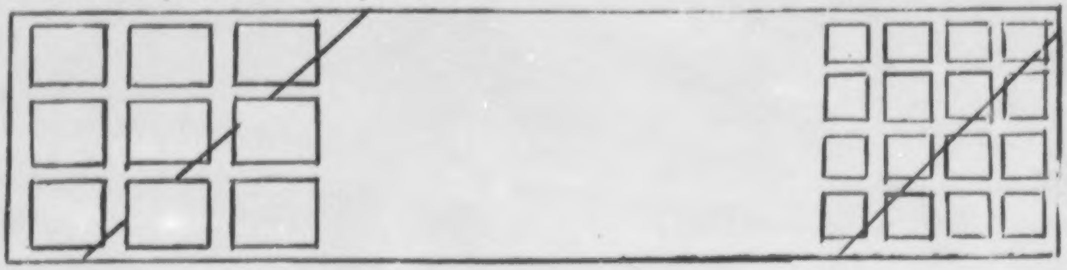
```
                                  X
                       X         X X
               X      X X       X X X
        X     X X    X X X     X X X X
  X    X X   X X X  X X X X   X X X X X
  1     3      6       10         15
```

```
                 1 = 1
             1 + 2 =  3
         1 + 2 + 3 =  6
     1 + 2 + 3 + 4 = 10
 1 + 2 + 3 + 4 + _ = __
```

See if the children can tell how the equations develop from the above drawings. The equation 1 + 2 + 3 = 6 goes with which drawing? By looking at the drawings, what would you name the set of numbers 1, 3, 6, 10, and 15? Many of the children will say triangular numbers because triangles are used in the drawings. Can you name the next triangular number? The child who mentions 21 should tell how he knows thatis the right answer. From the discussion, children will point out that the pattern is always to add the next counting number.

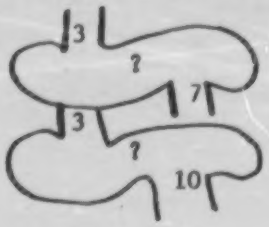
```
             *  1               *  1
             *  3               *  4
             *  6               *  9
             * 10               * 16
             * 15               * 25
TRIANGULAR   * 21       SQUARE  * 36
 NUMBERS     * 28      NUMBERS  *  .
             *  .               *  .
             *  .               *  .
             *  .               *  .
             *  .               *  .
             *                  *
```

Have the children look at the above set of triangular numbers and square numbers and see if they can find a pettern that connects them. Some children will see thatn when you add 1 and 3 the sum is 4; with 3 and 6 the sum is 9; with 6 and 10 the sum is 16. Adding two cinsecutive trinagular numbers gives a sum that is a square number. See if the children can show this is true by drawing pictures of triangular numbers and putting them toegther to form pictures of squares.



A game called "Guess My Rule" is another good way to get children to look for a pattern. After the children find the rule, have them state it--perhaps even write it down--in general terms.

START	ANSWER
2	7
3	10
5	16
7	?
N	?



WHAT IS THE RULE?

Put 2 in the machine, and out comes 7. Ask how the machine is fixed to produce this answer. If the children say that it is by adding 5, have htem put in 3 and have the machine add 5. The answer is 8, which does not agree with the result in the table. Therefore, you must go back and find out how the machine was fixed to produce 7 when you put 2 into it. Perhaps now a child will say the machine multiplies 2 by 3 and adds 1, which gives 7. Then they must try 3, multiplying it by 3 and adding 1 and see if the result of the manipulation is 10.

It seems they have found the rule, have the children make up rules for the other children to solve. To be sure, try it on the third example and see if, when you put in 5, the result is 16. Now you are sure the rule is multiply be 3 and add 1. What is the result if you put in 7? The children write 22 in the table. If you put N in the machine, the answer is 3 x N + 1 which is the rule in gereral terms.

After you have given several problems for which you derermine the rule, have the children make up rules for the other children to solve. Besides looking for a pattern or a rule, the children get lots of practice in adding, multiplying, substracting, and dividing.

In the middle grades, when children find patterns in base ten numerals, they should look for similar patterns in other bases. For example, most sutdents in the middle grades have discovered or have been told that the sum of the digits of the products of the nine-table is nine. Look at the products and you can see that the sum of 3 and 6 is 9, the sum of 5 and 4 is 9, etc. The sum of the digits of the products equals 9.

Try this: "If this same pattern occurs in base five, which table would you check?"

The answer is the four-table because four is one less than the base, which is five. In base ten, the pattern occurred in the nine table which is one less than the base, ten.

1 x 9 = 9
2 x 9 = 18
3 x 9 = 27
4 x 9 = 36
5 x 9 = 45
6 x 9 = 54
7 x 9 = 63
8 x 9 = 72
9 x 9 = 81

The pattern does work in other number bases, as you can see by the examples in base five and seven. The sum of the digits in the four table of base five is four, and the sum of the digits in the six table of base seven is six. There is another pattern in each of the number bases when you look at the products: digits in the ones place get smaller by one and digits in the base position get larger by one.

BASE FIVE	BASE SEVEN
1 x 4 = 4	1 x 6 = 6
2 x 4 = 13	2 x 6 = 15
3 x 4 = 22	3 x 6 = 24
4 x 4 = 31	4 x 6 = 33
	5 x 6 = 42
	6 x 6 = 51

Children are seldom given problems that require some thought and logical reasoning. If all of the problems they are required to solve have just one standard answer, children will never learn to think for themselves. Give them some alternatives.

FIVE "NONTRIVIAL" NUMBER GAMES*

Terrel Trotter, Jr.

Number games can provide elementary students with a good opportunity to test their observation skills for problem solving, and at the same time they will be practicing such drill skills as adding and subtracting. Here are five trivial number games guaranteed to provide hours of fascination and lots of hidden drill. The word trivial is used here with two meanings in mind. The games are trivial in the usual mathematical sense--namely, once the secret is known to both players, the game's winner is predetermined, based on the starting number and who begins play. The games are non-trivial from the point of view that they are valuable teaching aids for motivating student thinking.

The rules for the games will be presented first, followed by discussions of the respective winning strategies. You are encouraged to try the games before reading the solutions section. This way you may have the pleasure of discovering the secrets for yourself before challenging your students to attempt the same.

RULES

Game 1. The first player selects any integer from 1 to 10. Then the two players alternately add any integer from 1 to 10 to the sum left by the opponent. Play continues until one player can make an addition giving a grand total of 100. That player is thereby declared the winner.

Game 2. The rules are the same as those for game 1, except that now the winner is the player who forces his opponent to make the total 100 or more.

Game 3. The name of this game is "Aliquot." It was devised by David L. Silverman and appeared in the problem section of the Journal of Recreational Mathematics. Here is Mr. Silverman's own description from that journal (1970-a-):

> Two players start with a positive integer and alternately subtract any aliquot part (factor) with the exception of the number itself from the number left by the opponent. Winner is the last player able to perform such a subtraction.
>
> By way of example, if the original number is 12, first player may subtract either 1, 2, 3, 4 or 6 (but not 12). If he subtracts 2, leaving 10, second player may subtract 1, 2 or 5.
>
> The objective is to leave your opponent without a move. This can only be done by leaving him a 1, since 1 is the only positive integer with no aliquot part other than itself.

Game 4. This game, called "Proper Aliquot," was also devised by Mr. Silverman. In the same issue of the Journal of Recreational Mathematics, he gives the rules thus:

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*The Arithmetic Teacher: November, 1972, pp. 558-560.

> The rules are the same as those of Aliquot with the exception that only proper divisors may be subtracted. Consider 1 an improper divisor.

Game 5. The first player selects any reasonable large number. Then the two players alternately subtract any number they choose from the number left by the opponent, provided the chosen number meets one requirement: the number subtracted must not exceed twice the value of the number subtracted by the opponent on the previous play. For example, if player A subtracts 4, then player B can select any integer from 1 to 8. The game is won when one player can "take it all," that is, leave 0.

SOLUTIONS

Game 1. After playing this game a time or two, most students realize that whoever makes the total 89 can force the win. A player does this by adding the difference between 11 and whatever his opponent adds next. That is, if A adds 3 to 89, B adds 11 -3, or 8, making 100 and winning.

Similar reasoning leads to the conclusion that 78 is the next desirable total to obtain, since 78 = 89 -11. A continuation of the same reasoning yields the winning sequence, 67, 56, 45, 34, 23, 12 and 1. (An easy way to remember these numbers is to recognize that the tens digit is one less than the units digit.) Therefore, by starting with 1, the first player can force the win, regardless of what the other player does. At that point the game becomes trivial. (Of course, if a player doesn't know the secret, his opponent can conceal it from him longer by simply "entering" the sequence at some later point.)

Game 2. Inasmuch as 99 is the largest total a player can make without leaving or exceeding 100, it and the sequence derived by subtracting 11's are the secret totals. (Naturally, they are the multiples of 11.) Therefore, by allowing his opponent to select the first number, a player can once again force the win.

Game 3. The proper way to analyze this game is also to begin at the end. Only now the end is the integer 1. First, examine all the possible subtractions for some integers near the end of the game, say 2 through 10:

$$\begin{array}{r}2\\-1\\\hline 1\end{array}\quad\begin{array}{r}3\\-1\\\hline 2\end{array}\quad\begin{array}{r}4\\-2\\\hline 2\end{array}\quad\begin{array}{r}4\\-1\\\hline 3\end{array}\quad\begin{array}{r}5\\-1\\\hline 4\end{array}\quad\begin{array}{r}6\\-3\\\hline 3\end{array}\quad\begin{array}{r}6\\-2\\\hline 4\end{array}\quad\begin{array}{r}6\\-1\\\hline 5\end{array}$$

$$\begin{array}{r}8\\-4\\\hline 4\end{array}\quad\begin{array}{r}8\\-2\\\hline 6\end{array}\quad\begin{array}{r}8\\-1\\\hline 7\end{array}\quad\begin{array}{r}9\\-3\\\hline 6\end{array}\quad\begin{array}{r}9\\-1\\\hline 8\end{array}\quad\begin{array}{r}10\\-5\\\hline 5\end{array}\quad\begin{array}{r}10\\-2\\\hline 8\end{array}\quad\begin{array}{r}10\\-1\\\hline 9\end{array}$$

If a winning number is defined to be one from which a player can force a win, even with the best play from his opponent, a loser will be a number from which a player can only leave a winning number to the opponent, no matter what factor is subtracted. Thus, 2 is a winner--the ultimate winner, of course, since it leads directly to the game's objective. On the other hand, 3 is a loser because it leaves 2 to the opponent. Though 4 has two possible subtractions, it still can be classified as a winner because the best play (the factor 1) leaves a loser.

Further investigation along this line yields the conclusion that all even numbers are winners and all odd numbers are losers. This suggests a simple rule for winning: If player A faces an even number, he subtracts one of its odd factors so that he leaves an odd number for his opponent to play from. The logic behind this is that the only factors of an odd number are also odd, and odd-odd = even. Therefore, the opponent must return some other even number, and the cycle is established. Eventually he must leave the 2, and then player A wins.

Game 4. The exclusion of 1 from the set of factors makes some interesting changes in the strategy. Now all the primes are the objectives of the game and therefore are immediate losers. Testing the other numbers as was done for game 3 leads to the conclusion that all odds are still losers, but not all evens are winners. For example, 8 is the first even loser--after 2, that is, which is also prime. Identification of the losing even numbers becomes less difficult when the next even loser (32) is found. Factoring these three even losers yields 2^1, 2^3, and 2^5. These are odd powers of 2, that is, 2^n where n is odd.

In summary then, the winning procedure is to subtract such factors that leave either an odd number or an odd power of 2.

Game 5. The secret behind this game is a great deal more sophisticated and elusive than the secrets behind the other four. It relies on some high-powered concepts from higher mathematics. But, the secret is relatively easy to learn and use.

A preliminary observation about the game should be noted. A player should never subtract a value that is one-third or more of the number. If he does, the opponent can always take all the remainder. For example, if a player faced 17 and subtracted 6, leaving 11, his opponent could "take it all" because the opponent can go as high as 12.

The Fibonacci numbers are an essential part of the secret. Simply stated, the Fibonacci numbers are integers from the sequence 1, 2, 3, 5, 8, 13, 21, ..., where succeeding members are found by adding the two preceding members. A theorem from number theory, the Zeckendorf theorem, says, in effect, that all non-Fibonacci numbers can be represented uniquely as the sum of two or more nonconsecutive Fibonacci numbers. (For a proof of this theorem, see Hoggatt -1969, p. 74-.) Here are a few examples to illustrate this concept:

(a) $17 = 13 + 3 + 1$

(b) $29 = 21 + 8$

(c) $40 = 33 + 5 + 2$

It turns out that when playing from a non-Fibonacci number, a player can force the win by subtracting the smallest Fibonacci number appearing in the representation. This succeeds as a result of the easily proven fact that for any two non-consecutive Fibonacci numbers, the smaller one is always less than one-half the larger. The significance of this is that it always allows the player to give his opponent a Fibonacci number at some future stage of the game. From there, the opponent must then return a non-Fibonacci number to the player. It will have a new representation with a smalles Fibonacci addend, and the cycle is repeated.

An illustration will serve to clear up the preceding discussion. Playing from 17 which equals 13 + 3 + 1, player A will take the 1. Now B can only subtract 1 or 2, neither of which will take all the 3 in the representation. Whatever B subtracts, A will then take whatever is left of the 3. This presents B with the Fibonacci number 13. The greatest possible number B can now subtract is 4, depemding on how the two previous subtractions were made. In any case, A will have a new non-Fibonacci number to play from, and he can return to the winning procedure described above.

As mentioned in the beginning, these games are indeed trivial from a pure, mathematical standpoint. Obviously, if both players know the secret, there would be no point in playing a game. The first to play would hand the other a losing number, and for all practical purposes the game would be over right away. If only one player knew a game's secret, he could always win too, much to the consternation of his patsy. But, if both players are unaware of the secrets, the games provide a lot of practice in basic skills. The importance is that the drill is only a means to and end, not an end to itself.

In order to enjoy mathematics, students need to experience some success along the way. These games often allow the poorer student to achieve some wins over the better student, especially at first, before the strategies are either discovered by the students themselves (it is hoped) or revealed to them. The more talented student shouls be encouraged to seek the strategies as exercises in true problem solving. It is recommended that the secrets not be revealed too soon, it at all. Telling the secrets of the games destroys the fun and denies an individual the satisfaction of finding them for himself. Either way, the games are far from trivial as learning experiences.

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________________ (b) Your Move. New York: McGraw-Hill Book Co., 1971.

MATH LAB ACTIVITY: ARROWMATH*

Children of all ages (adults too) seem to love anything that resembles a code game. "Arrowmath" combines a code game with computation drill.

The activities in this article (which can be duplicated as is for use in your math lab) are arranged in order of difficulty, with the first ones requiring only addition and subtraction, and the final group utilizing all operations.

The result for some students will be some solid experience in computing. For others that are more creative, they will have the opportunity to invent their own math system and have a chance to discover patterns.

On with "Arrowmath":

ARROWMATH ON A GRID

Fill in the grid below using the following rules:

↑ means + 2

→ means + 5

↓ means - 2

← means - 5

Start with 14 and write in the numbers according to the moves from 14, keeping in mind the four rules cited above. (For example, 14 → means 14 + 5. Fill in 19 in the grid just to the right of the 14.)

		14		

Look at the grid now that you have filled it in and tell what the following mean:

↗ means ________________

↙ means ________________

↘ means ________________

↖ means ________________

---

*GRADE TEACHER, March, 1972; pp. 38-40.

Make up another 5 by 5 grid and change the rules:

↑ means + 4

← means + 7

↓ means - 4

→ means - 7

Start at 36 and write in the numbers following the same procedure as before.

		36		

After you have completed the grid, tell what the following mean:

↗ means ____________________

↘ means ____________________

↙ means ____________________

↖ means ____________________

Design some grids of your own. Make up the rules and let your friends fill in the grids.

100 CHART AND ARROWMATH

Look at the chart (right) and tell what the following arrows mean:

→ means ____________________

↑ means ____________________

↖ means ____________________

← means ____________________

↗ means ____________________

↘ means ____________________

↓ means ____________________

↙ means ____________________

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Now you will use the arrows to get from one place to another on the chart. For example, 54 ↑ → ↘ is the same thing as 54 - 10 + 1 + 11 = 56. Complete the following:

1. 13 → → → is the same thing as ______________________ = ________.
2. 77 ↑ ↑ is the same thing as ______________________ = ________.
3. 25 ← ↓ ↓ is the same thing as ______________________ = ________.
4. 52 ↓ ↑ ↑ → is the same thing as ______________________ = ________.
5. 76 ↓ ↑ ↓ is the same thing as ______________________ = ________.

Now, look for the short cuts:

6. 25 ← ↓ ↓ is the same thing as what two arrows? ______________________
7. 80 ↑ ↓ ↑ ↓ ↑ ↓ ↑ is the same as what? ______________________
8. 61 → → ↑ ← ← is the same as what? ______________________

Make up some of your own and try them on a friend.

Still using the 100 chart, reverse the operation, that is, list two places and fill in the arrows for getting from one to the other. For example, "Get from 54 to 56 with three arrows." (Answer: 54 ↑ → ↘). Now solve these:

1. Go in five arrows from 12 to 67. (12 ______________________ =67.)
2. Go in three arrows from 82 to 73. (82 ______________________ =73.)
3. Go in two arrows from 74 to 66. (74 ______________________ = 66.)
4. Go in four arrows from 26 to 19. (24 ______________________ = 19.)
5. Go in five arrows from 100 to 86. (100 ______________________ = 86.)

Make up some paths for your friends. Be tricky!

HEXAGON MYSTERY

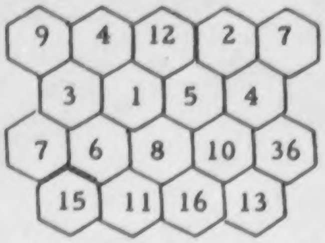
Follow these rules:

→ means add the number to the right.

← means subtract the number to the left.

↗ means multiply the number above and to the right.

↖ means divide the number above and to the left.



Use the hexagons to solve the problems below. (Examples: 1 → = 6; 5 ← = 4; 6 ↖ = 2; 1 ↗ = 12; 1 → → = 10.)

1. 6 → = ________.
2. 6 → → = ________.
3. 3 ↗ → = ________.
4. 15 → ← = ________.
5. 8 ↗ ← = ________.
6. 8 → ↗ = ________.
7. 11 → ↗ = ________.
8. 14 ↖ → = ________.
9. 10 ← ↗ → ↖ = ________.
10. 3 ↗ → → → = ________.
11. 16 ↖ ↗ → ↖ → = ________.

Make up problems for your friends to solve. Or, invent a math system of your own. If you don't want to use hexagons, try squares. Make up different rules for the arrows.

(Of course, if you're really clever and want to save yourselves work in making up the grids for all the activities, just take a calendar for any month and play "Arrowmath" with that. Try it........it works!)

NUMBERS HAVE MANY NAMES

A very powerful and fundamental mathematical idea is the one that any given number concept may be identified or named by a host of numerals or number names. The mathematician chooses those names or numerical representations of number that are most convenient for his particular solution. Encouraging this style of thinking will allow student to stir his mathematical conjectures.

To develop the activity "Numbers Have Many Names," begin with rather simple renaming such as how many ways can you name 6?

Names for 6

3 + 3   6 + 0   7 - 1   8 - 2

3 X 2   2 + 2 + 2 $\frac{1}{2} + 5\frac{1}{2}$

18 ÷ 3 $\frac{24}{2} \times \frac{1}{2}$

To extend the renaming game, try the following:
Start with the digit 1 and successively double each numeral 1,2,4,8,16 etc. Use only these numerals to count in sequence. Add more numerals by doubling when needed. Each digit is to be used only once in each renaming:

1 - 1
2 - 2
3 - 1+2
4 - 4
5 - 4+1
6 - 4+2
7 + 4+2+1

8 - 8
9 - 8+1
10 - 8+2
11 + 8+2+1
12 - 8+4
13 - 8+4+1
14 - 8+4+2

etc.

Can you find any counting number that cannot be name with these digits? Try counting in sequence using only odd numbers; how far can you go? Can you think of other kinds of digits which you can use to invent another way to count in sequence?

PICK A DATE:

Pick a date from history. How far can you count using only its digits? Try the date 1932--the date of the election of Present Franklin Roosevelt. Use all the digits in this date, one time each, to count in sequence. Here is another illustration of the many ways in which a number concept--in this case, 1932--can be identified or named:

1 = 1 x 9/3 - 2   2 = 9 - 1 - (3x2) $3 = \frac{1+9}{2} - 2$ 4 = 9/3 - 1 + 2

5 = (2x9) - 13   6 = (1 x 9/3) x 2   7 = (9/3 x 2) + 1   etc.

Numbers Have Many Names

HOW MANY NAMES?:

How many names for each digit 1 through 10 can you find using only digits of less value?

1 → none
2 → 1+1 1 name
3 → 1+1+1
1+2 (2+1 is the same)
2 names
4 → 1+1+1+1
1+1+2 4 names
2+2
3+1
5 → 1+1+1+1+1
1+1+1+2
1+1+3 6 names
1+4
2+2+1
2+3

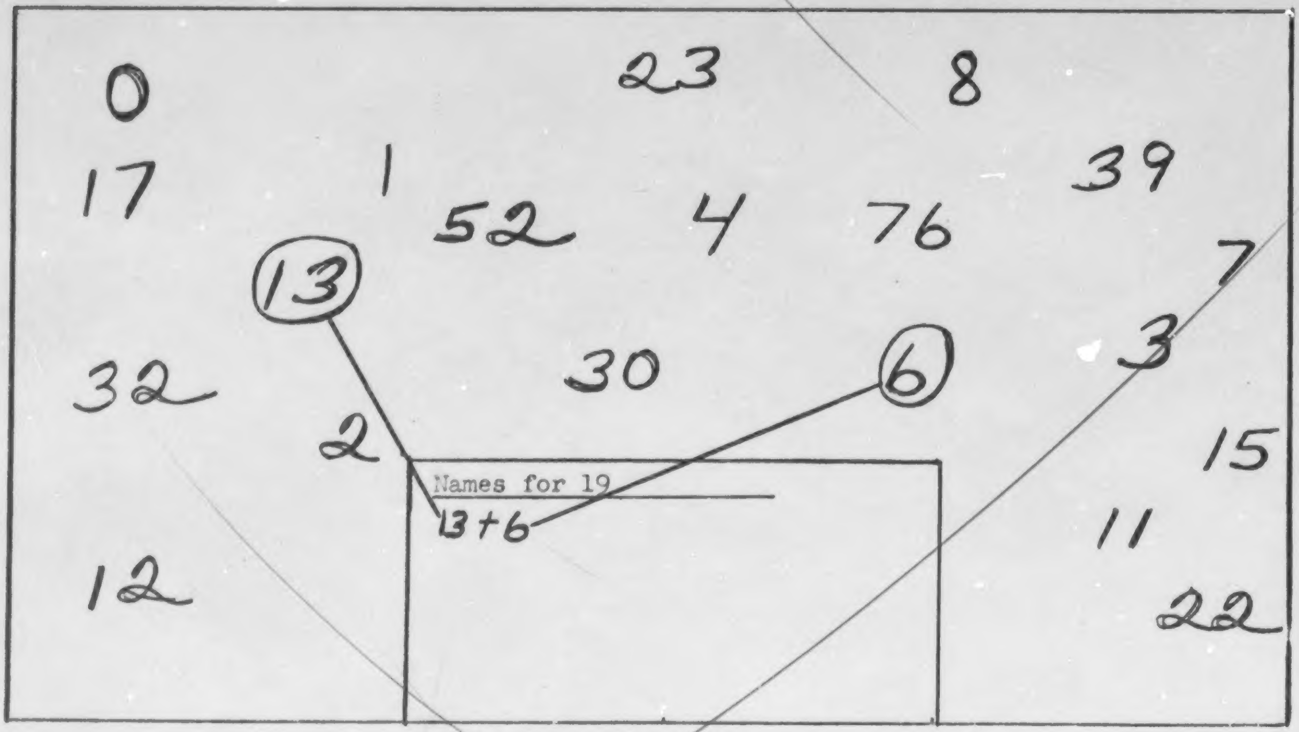
6 → 1+1+1+1+1+1
2+1+1+1+1
2+2+1+1
2+2+2 10 names
3+1+1+1
3+2+1
3+3
4+1+1
4+2
5+1
7 = 14 names
8 = 21 names
9 = 29 names
10 = 39 names

Can you find each of the names for 7,8,9 and 10?

NUMBER NAME HUNT:

Develop a number-name hunt such as the following:

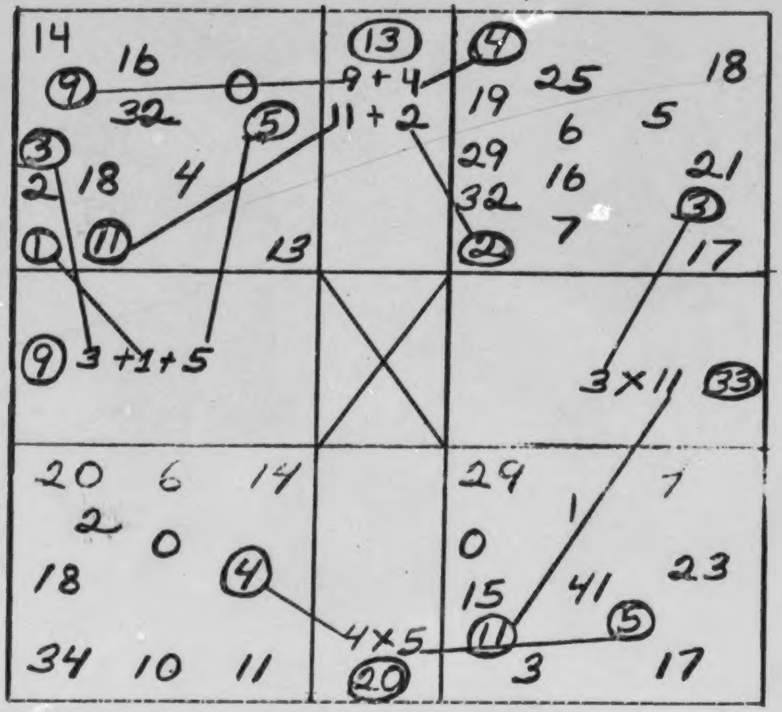
Directions: Choose from among the various numerals and use any operation signs to find names for 19. Record in the 19 box.



Numbers Have Many Names

EXTENDED NUMBER NAME HUNTS:

Directions: Find names for numerals in circles from adjacent corners only. How many can you find?


14
16
9
0
32
5
3
2
18
4
1
11
13
13
9+4
11+2
4
25
18
19
5
6
29
21
16
32
3
7
2
17
9
3+1+5
3×11
33
20
6
14
2
0
18
4
34
10
11
4×5
20
29
7
1
0
23
15
41
5
11
3
17

A GAME FOR REVIEWING BASIC FACTS OF ARITHMETIC *

Edward E. Arnsdorf

A delightful game to use in reviewing addition-subtraction and multiplication-division facts for whole numbers has been developed by a Sacramento State College education student, Mary Dunlap.

Materials for the game consist of forty-five cards cut from tagboard. Three-by two inch cards work well. The tagboard cards are numbered as shown in Figure 1. Two different colors should be used for the numerals on the cards. With all top numerals one color and all bottom numerals another color, the cards can be arranged easily, right side up.

The game may be played by two, three, or four persons. To begin play, the deck is shuffled and five cards are dealt to each player. One card is turned face up and the remaining undealt cards are placed in a stack face down. The person at the immediate left of the dealer plays first.

Suppose the (6, 2) card (the one with the 6 on top and the 2 on the bottom) is the one that has been placed face up. The numbers on this card may represent 6 + 2 = 8, 6 - 2 = 4, 6 x 2 = 12, or 6 ÷ 2 = 3. If the first player has a card containing two numbers whose sum, or difference, or product, or quotient is 8, or 4, or 12, or 3 respectively, he may play the card from his hand by placing it face up on the (6, 2) card. If he does not have such a card in his hand, he must draw the top card from the pack and then determine if it can be played. If the drawn card cannot be played, it is added to the player's hand and he draws again. The process of drawing from the pack continues until an 8, or 4, or 12, or 3 is found. When a card can be played, it is placed on the face-up pile and the next player takes his turn. For example, assume that the first person plays the (8, 2) card, representing 8 ÷ 2 = 4, on the opening (6, 2) card. The second player must now determine the new numbers that can be played. He may use any one of the four operations to name one of the new numbers, in this instance either 10 or 6 or 16 or 4.

Anytime a card is played, another player may challenge the play by asking why that particular card can indeed be played. The game is won by the first person to play all his cards. If all the cards have been drawn and the game has not been won, all but the top card on the face-up, or discard, pile are reshuffled and the game continues.

Notice that the two numbers on some of the forty-five cards cannot be used to name four different whole numbers. The sum and the product may name the same number; or the product and the quotient may name the same number; or the operation of division may not yield a whole number. Since some cards are more likely to be playable than are others, there are opportunities to make strategic decisions.

Players of this game are continually mentally performing the operations of addition, subtraction, multiplication and division. Hence this game can be both an

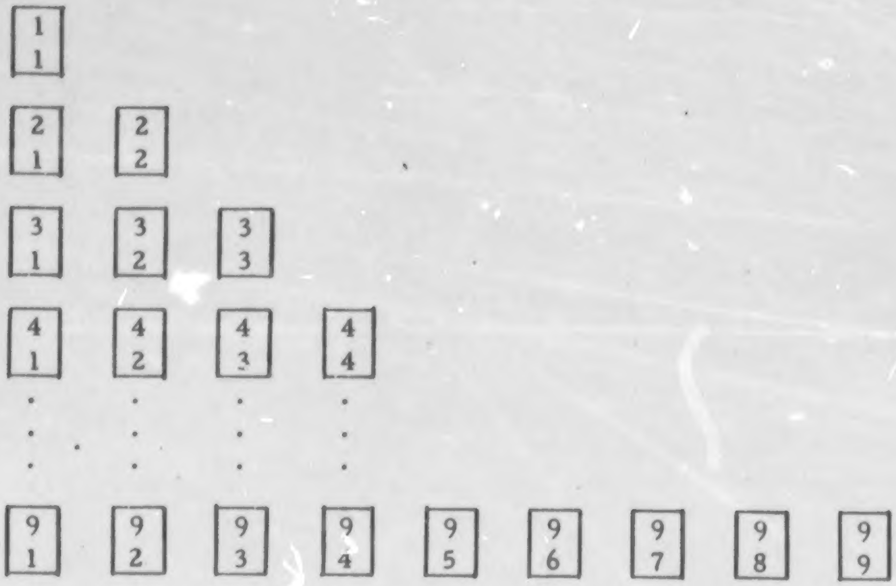

1 1
2 1
2 2
3 1
3 2
3 3
4 1
4 2
4 3
4 4
9 1
9 2
9 3
9 4
9 5
9 6
9 7
9 8
9 9

Figure 1

effective and an enjoyable way to review basic facts of arithmetic.

A STUDENT-CONSTRUCTED GAME FOR DRILL WITH INTEGERS*

Merle Mae Cantlon
Jefferson Junior High School Caldwell, Idaho

Doris Homan
Vallivue Junior-Senior High School Caldwell, Idaho

Barbara Stone
Central Junior High School, Nampa, Idaho

In teaching a unit on integers, teachers are invariably faced with the age-old problem of how to provide interesting and meaningful drill. Popsicle sticks can be used to provide a motivating, "hands on" laboratory exercise in which the students build their own game. The anticipation of using what they are studying in a fun situation seems to be an added motivational factor for students. The exercise shown in figure 1 is a suggestion for the first activity, in which the children make the materials and experiment with the "Popsicle-Stick Game."

For this activity, the students should be divided into groups of three or four. Without further instructions, each group is given its assignment sheet for the day (fig. 1).

The second activity involves the actual playing of the Popsicle-Stick Game with the numbered Popsicle sticks that the children have made. Instructions for playing the game follow:

1. The class is divided into teams of three or four students, and each team is given a set of numbered Popsicle sticks.

2. Students on each team take turns acting as captain, the honor rotating around the table with each new problem. A permanent scorekeeper should be chosen for each team.

3. The object of the game is to see which team, using one or more of the four operations on integers, can be the first to construct a mathematics problem with an answer equal to the integer that the teacher has written on the board. Numbers from -100 through 100 can be used for the integer answers, and for variation, these can be written on slips of paper and then drawn from a box by the captain of the last winning team.

4. When a team has constructed a problem with an answer equal to the integer on the board, all members of the team must raise their hands. The teacher notes the first team whose hands are all raised. When it is apparent that most groups have completed a problem, time is called.

5. The completed problems are written on the board (or on the overhead projector) by the respective team captains, and the accuracy of all problems is checked by the class. The team captains have the responsibility for writing the correct operations as well as the correct integers, and for indicating the order in which

Building a Popsicle Stick Game

Names of team members:____

Check the box at the right when the team has completed each step, please.

1. Get 15 Popsicle sticks and one felt marker. ☐

2. Mark -1 on the first stick, keeping the sticks in vertical position. Mark -2 on the second stick, and so on through -5. ☐

3. Mark 0 on both sides of one stick. ☐

4. Mark -1 on one side of a stick and 1 on the opposite side. Mark -2 on one side of a stick and 2 on the opposite side, continuing on in this manner through -9 and 9. ☐

5. Build with Popsicle sticks the following math phrases (expressions) that are equal to -4.

   a. -3 + -1 = ☐

   b. -8 + 4 = ☐

   c. (-4 x 2) + 4 = ☐

6. With your sticks, write three math phrases that are equal to 48.

   a. ____________________=48 ☐

   b. ____________________=48 ☐

   c. ____________________=48 ☐

7. Using at least one negative integer in each, write three math phrases equal to 3.

   a. ____________________=3 ☐

   b. ____________________=3 ☐

   c. ____________________=3 ☐

Figure 1.

5. continued......
the operations are to be performed.

6. If the problem prepared by the first team whose hands were raised is correct, the team receives four points. Two points are awarded to each of the other teams

6. continued.....
that has a correct problem. Teams with incorrect problems receive a score of -1. Each team keeps its own score.

7. For variations, the teacher may place restrictions on the problems to be constructed. For example, some possible restrictions are:

   a. There must be at least one negative integer in the problem.
   b. At least two operations must be used in the problem.
   c. The problem must be worked with a minimum of two sticks, or three sticks, and so on.

The Popsicle-Stick Game has several advantages. It promotes total student participation--groups are small. The honor of acting as captain is rotated within the groups. All members of the team must raise their hands in order for the team to be recognized as first. All students are involved in checking the problems that are put on the board, since numerous correct responses are possible. The scoring procedure provides additional opportunities for practice. "Hands on" materials are used, and little teacher direction is needed.

In summary, this experience provides students with useful and enjoyable practice for operations with integers through the use of manipulative materials in a laboratory setting. In the words of a Chinese philosopher: I hear and I forget; I see and I remember; I do and I understand.

EDITOR'S NOTE: I assume that there has been previous development of understanding of directed numbers and that these activities will provide practice, motivation, and further understanding. The use of manipulative materials should be most helpful as another means for symbolizing exercises, and the group involvement provides encouragement and support for children not quite sure of the meaning of directed numbers.

TUF GAME

T U F....................TUFFER....................TUFFEST

(A "Superlative" Mathematics Game)

What is TUF?:

TUF is a mathematics game based on a number sentence or equation.

TUF is a series of games:
Initial games involve only the operations of addition and subtraction.
The game progressively advance by adding more mathematical operations and concepts:

multiplication	negative numbers
division	number systems to other bases
parentheses	ratio
fractions	proportion
decimals	percentage

(NOTE: Still more complex games are possible with TUF: exponential powers, fractional roots both positive and negative, logarithms, tangent, pi, i or j the $\sqrt{-1}$, and an unknown X.)

GAME EQUIPMENT:

20 - RED Operator Cubes marked: = + - x ÷ (x) -
12 - BLUE Number Cubes marked: 0 1 2 3 4 5
12 - GREEN Number Cubes marked: 6 7 8 9 10 ⊙
8 - YELLOW Fraction Cubes marked: 1/2 1/3 1/4 1/5 1/8 1/10
4 - WHITE Equals Cubes marked: = = = = = =
4 - ORANGE Constant Cubes: % log j tan X π
4 - BLANK Cubes
3 - TIMERS: RED - 3 min., YELLOW - 2 min., BLUE - 1 min.
1 - Set Rules
1 - Game Container

RULES FOR TUF:

There are a number of TUF games that can be played, each based on particular mathametical operations. It is suggested that players start with the BASIC GAME, become familiar with the play, then move on to the game that interest one. If you are not familiar with the mathematics of a particular game, you will find the examples accompanying each game sufficient for playing purposes.

GAME 1 -- BASIC GAME:

2 to 4 people OR teams can plan Game 1.
Each player or team takes: 3 BLUE number cubes. 4 RED operator cubes
3 GREEN number cubes. 1 WHITE equals cube

On a signal from one player (or preferrably a game manager), each player rolls his cubes out on the table from his hands. The UPPER FACE of the cube is used -- the cube CANNOT BE TURNED after the throw.

TUF GAME

TUF....................TUFFER...................TUFFEST

The first person to form an equation, using as many cubes as he can, CALLS OUT, "TUF!" For typical equations, see the examples below:

After DECLARING his equation, TUF starts a timer and removes his hands from the table. The other players are allowed this time interval to try to form a longer equation than TUF's. TUF has this time to mentally try to think of a longer equation than the one he has. If one of the other players can form a longer equation, that is use more cubes than TUF's in the allotted time after TUF DECLARES, he calls out, "TUFFER!" and sets the second timer. TUF than has an opportunity with the other players to try to exceed the length of TUFFER'S equation during the second timer interval. He cannot change the face of the cubes by turning them, but he may re-arrange his cubes in any order he wishes. If anyone succeeds in beating TUFFER, he calls, "TUFFEST!" and sets the next timer to allow the other players to review their equations and attempt to improve them -- that is, make them larger. Again, the players cannot change the face of their cubes, but may re-arrange their cubes in any order they wish to form a valid equation. The game may continue ("MORE TUFFEST," " MOST TUFFEST") until either a player uses all his cubes or the alloted time interval runs out. The BLUE timer is used again -- the fractional remaining time is allowed to run before resetting.

The FINAL DECLARER (with a correct equation) SCORES 2 points for each cube used including the equals cube. The equations formed by the other players are counted by allowing one point for each cube used. If a player is found to have an incorrect equation OR if no equation is completed, he counts ZERO for that round.

At the end of the time interval, the scores are calculated for that round and noted on a score sheet. The highest total score after 5 games is the winner. Totaling scores over a number of games is important. Players quickly learn the importance of making A SHORT EQUATION for scoring purposes even though they haven't a winner. No scores are calculated until some timer runs out -- this completes a round or game.

The suggested use of the timers is:
RED - 3 min. for TUF
YELLOW - 2 min. for TUFFER
BLUE - 1 min. for TUFFEST
For "faster games," use the timers as follows:
YELLOW - 2 min. for TUF
BLUE - 1 min. for TUFFER and TUFFEST.
for "even faster games," use the timers as follows:
BLUE - 1 min. for TUF, TUFFER, AND TUFFEST.

Any fractional amount of time left over is allowed to run and the timer is then reset by the DECLARER to give the full minute before "time" is called. Timers may also be combined to give longer intervals.

REMINDER: As soon as TUF DECLARES (or any other DECLARER), he must remove and keep his hands off the table. He CANNOT change his equation in any way, even if he sees an error or a longer equation. The other players must take their hands off the table at the end of the time interval when scores are calculated.

T U F....................TUFFER...................TUFFEST

VARIATION OF GAME I: (for TEAMS of PLAYERS:)

Follow the same general procedures described above, however, teams of 2 or more players form behind their respective playing position. Team members SHOULD NOT KIBITZ or help their team member player, however, the team members may study the faces of the cubes being used and hold CONFERENCES or STRATEGIES DISCUSSIONS. At no time should a team member hint to their team player of an error or longer equation. At any time, the GAME MANAGER may call, "SWITCH" and at that signal all teams must exchange their seated team player for one of the other team members. At each signal to SWITCH, each team must rotate a new team member to act as team player. In this way, all team members rotate in order. To increase the movement and pace of the game, the GAME MANAGER may also call, "SHAKE AND ROLL" at any time. At this signal, the seated team player picks up all of his cubes, shakes them, and then rolls them out again and proceeds with equation building. If this signal occurs during the time that someone has DECLARED, that person DOES NOT TOUCH his cubes and the timer continues to run. Scores now become team scores. Even the "weakest" of players can become involved.

An added variation to the basic game involves the "setting of a convention." Such a convention for a game or round might be to allow the rotation of one RED cube or one GREEN cube or one BLUE cube or one YELLOW cube. The GAME MANAGER must be alert to allow only one cube to be rotated according to the convention agreed upon. Since there are 4 BLANK cubes in the TUF game, one of these could be used by each player or team as a "WILD" cube. As a ground rule for this convention, the GAME MANAGER may record on a note card what the WILD cube is used for (this may be called out by the player), and until someone DECLARES or until a timer runs out, the player may not re-use the WILD cube except for the function it was identified. Many possible variations or conventions are possible. Let the players initiate some variation or convention of their own.

Equation Examples for Game I:

1. $\boxed{4}\ \boxed{=}\ \boxed{4}$
2. $\boxed{3}\ \boxed{-}\ \boxed{2}\ \boxed{+}\ \boxed{6}\ \boxed{=}\ \boxed{7}$
3. $\boxed{2}\ \boxed{-}\ \boxed{6}\ \boxed{-}\ \boxed{5}\ \boxed{=}\ \boxed{-}\ \boxed{9}$
4. $\boxed{4}\ \boxed{x}\ \boxed{3}\ \boxed{=}\ \boxed{6}\ \boxed{+}\ \boxed{6}$
5. $\boxed{10}\ \boxed{\div}\ \boxed{-}\ \boxed{5}\ \boxed{x}\ \boxed{3}\ \boxed{=}\ \boxed{-}\ \boxed{6}$
6. $\boxed{-}\ \boxed{2}\ \boxed{=}\ \boxed{-}\ \boxed{2}$
7. $\boxed{3}\ \boxed{x}\ \boxed{-}\ \boxed{2}\ \boxed{=}\ \boxed{-}\ \boxed{6}$
8. $\boxed{8}\ \boxed{x}\ \boxed{0}\ \boxed{=}\ \boxed{0}\ \boxed{x}\ \boxed{-}\ \boxed{3}$
9. $\boxed{10}\ \boxed{\div}\ \boxed{5}\ \boxed{x}\ \boxed{3}\ \boxed{=}\ \boxed{6}$
10. $\boxed{10}\ \boxed{-}\ \boxed{4}\ \boxed{\div}\ \boxed{2}\ \boxed{=}\ \boxed{8}$

T U F....................TUFFER....................TUFFEST

To give more variables and add interest, one number is allowed to be formed from two cubes:

11. $\boxed{4}\quad\boxed{x}\quad\boxed{7}\quad\boxed{=}\quad\boxed{2}\boxed{8}$

12. $\boxed{1}\boxed{5}\quad\boxed{-}\quad\boxed{8}\quad\boxed{=}\quad\boxed{7}$

13. $\boxed{3}\quad\boxed{x}\quad\boxed{10}\quad\boxed{=}\quad\boxed{3}\boxed{0}$

14. $\boxed{10}\quad\boxed{x}\quad\boxed{10}\quad\boxed{+}\quad\boxed{4}\quad\boxed{+}\quad\boxed{1}\quad\boxed{=}\quad\boxed{10}\boxed{5}$

Order of Arithmetic Operations: First do all MULTIPLICATION AND DIVISION in order from LEFT TO RIGHT. Then the ADDITION AND SUBTRATION is done in any order.

15. $\boxed{8}\quad\boxed{-}\quad\boxed{6}\quad\boxed{x}\quad\boxed{2}\quad\boxed{=}\quad\boxed{-}\quad\boxed{4}$

16. $\boxed{8}\quad\boxed{-}\quad\boxed{-}\quad\boxed{6}\quad\boxed{x}\quad\boxed{2}\quad\boxed{=}\quad\boxed{2}\quad\boxed{x}\quad\boxed{10}$

17. $\boxed{8}\quad\boxed{\div}\quad\boxed{-}\quad\boxed{2}\quad\boxed{=}\quad\boxed{-}\quad\boxed{4}$

18. $\boxed{10}\quad\boxed{-}\quad\boxed{10}\quad\boxed{\div}\quad\boxed{2}\quad\boxed{=}\quad\boxed{5}$

19. $\boxed{-}\quad\boxed{8}\quad\boxed{\div}\quad\boxed{-}\quad\boxed{2}\quad\boxed{=}\quad\boxed{4}$

20. $\boxed{-}\quad\boxed{8}\quad\boxed{\div}\quad\boxed{2}\quad\boxed{=}\quad\boxed{-}\quad\boxed{4}$

NOTE: For game 1, there are two multiplication signs: x and (x) on the RED cubes. There are two ZEROS: 0 on the blue cubes and ⊙ on the GREEN cubes -- a zero with a dot or decimal point in the middle. For game 1, the x and the (x) are both used for multiplication signs and the 0 and ⊙ are both used for zeros.

The positive or plus sign can only be used where normal mathematical usage requires it. REDUNDANT OR UNNECESSARY use of plus + is not allowed:

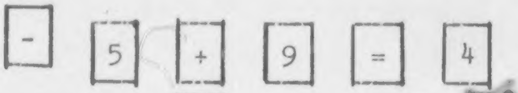
21. $\boxed{-}\quad\boxed{8}\quad\boxed{\div}\quad\boxed{-}\quad\boxed{4}\quad\boxed{=}\quad\boxed{2}$ NOT $\boxed{+}\quad\boxed{2}$

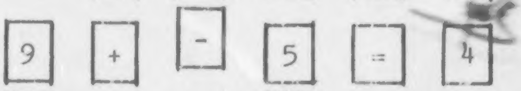
22. $\boxed{9}\quad\boxed{-}\quad\boxed{5}\quad\boxed{=}\quad\boxed{4}$ NOT $\boxed{+}\quad\boxed{9}$

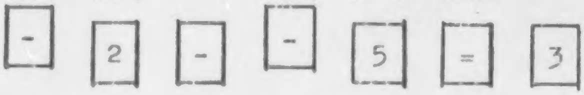
23. $\boxed{-}\quad\boxed{3}\quad\boxed{x}\quad\boxed{-}\quad\boxed{3}\quad\boxed{=}\quad\boxed{9}$ NOT $\boxed{+}\quad\boxed{9}$

TUF....................TUFFER....................TUFFEST

The NEGATIVE SIGN is indicated by raising the - cube slightly above the others. When SUBTRACT or MINUS is to be shown, the - cube is place in line with the number cubes:

24.


25.


26.


NOTE: Because EQUATING TO ZERO can become an OVER-USED GIMMICK, it can be disallowed if agreed on before the game starts, or discouraged with a two point penalty (two cube penalty) deduction. For instance, if TUF formed an equation with 11 cubes by equating to zero, his equation would only have a 9 cube value. TUFFER could then DECLARE with only ten cubes.

It might be helpful to identify any conventions or ground rules on a sheet of paper or on a chalk board before play commenced or at the point when a player raised a question about some procedure and a ground rule or convention was applied. Keeping the rules open and flexible makes for an interesting game as well as making it possible for players to modify the rules to fit their own playing styles and expertese. The sharper the players, the moreconventions or ground rules which cause them to createmore complex equations.

GAME I-A -- TUF for Very Young Players:

For pre-school and grades 1 and 2 children, TUF can be played using ADDITION and SUBTRATION operations ONLY. This primary game can be played in several ways:

(a) Using the RED operator cubes as constants, they may be set at either 4 + or 2 + or 1 + or 2 - or 3 - or 4 - etc.
(b) Rolling the RED operator cubes with the number cubes as in the BASIC GAME but before the play begins, change the multiplication signs X and (X) to addition +, and the division signs ÷ to subtraction -.

GAME I-B -- CUTTHROAT TUF:

For hardened veterans of the TUF arena, a more agressive and competitive version can be played by making a small change in the system of scoring. When the DECLARER of the moment TUF, TUFFER, or TUFFEST is bested by another player, his score reverts to zero instead of to a single count. He can score again only by making a new equation with a count equal to TUFFER (or TUFFER to TUFFEST), during the allotted time interval. He then counts a single score. Or better yet, he can beat the new declarer and call TUFFEST (or TUFFEST AGAIN) and return to his double score position.

If TUF, TUFFER, or TUFFEST have an INCORRECT EQUATION and the ERROR is recognized by another player, he can call out, "TUFF ERROR", and if all agree that TUF is in error the discoverer scores the equivalent of all the blocks used by the incorrect declarer. On the other hand, if the error discoverer is found to be the one in error, he deducts a like amount.

T U F....................TUFFER....................TUFFEST

This method of scoring injects a new element of risk (and caution!) which adds spice to the game for experienced players. It also tends to temper the impetuous; quick thinking player and encourages longer, more complicated equations.

GAME I-C -- SOLITAIRE:

Several versions of solitaire can be devised:

(a) For a given roll of cubes, the goal is to make the longest equation possible in a given time, such as five minutes.

(b) Larger numbers of blocks could be used: 4 BLUES, 4 GREENS, 6 REDS, and 2 YELLOWS with a longer interval of time allowed such as 15 minutes. The special incentive points (see GAMES 2 and 6) could be used to encourage more difficult equations.

(c) For a type of double solitaire for two players, one set of cubes is rolled out. A second set is required for the second player and these cubes are set to duplicate the original roll. Then on the word "GO", each player tries to create the longest (or highest point score if incentive points are used -- see GAMES 2 to 6) equation in a given time: 2, 4, or 6 minutes. The advantage of using the incentive, or bonus points, is that a player is not sure if he has a winning score even when he incorporates all the cubes into equations.

FOR FURTHER INFORMATION:

1. Make a set of TUF cubes and play the game.

2. See THE TUF GAME instruction book for other variations.

GAME OF FREEBLES

Game surface: eleven congruent square regions: (marked on wood or tag board)

RED	RED	RED	RED	RED	OPEN	BLUE	BLUE	BLUE	BLUE	BLUE
5	4	3	2	1		1	2	3	4	5

FREEBELS:

Red Freebels -- 5 in all.

Blue Freebels -- 5 in all.

(NOTE: Colored construction paper squares of two colors, or two different colored poker chips, or coins of two different denominations may be used as Freebels.)

Game Play:

To begin the game, 5 Red Freebels are placed in the 5 squares at one end of the playing surface, and 5 Blue Freebels are placed in the 5 squares at the other end. The middle square region is left open.

Example:

R	R	R	R	R		B	B	B	B	B

The OBJECT of the game is to interchange the position of the Red and Blue Freebels in a minimum of moves.

Rules for moving a Freebel:

1. A Freebel can be moved "forward", from one square to the adjacent square.
2. A Freebel can be moved by "jumping" over a Freebel of the OPPOSITE COLOR.
3. Moves "backward" are not allowed. (It would no longer be a minimum number of moves if we reversed moves.)
4. Jumping a Freebel "DOES NOT REMOVE IT FROM THE PLAYING SURFACE."

GAME VARIATIONS:

Game 1: Place Freebels as shown above -- 5 Red and 5 Blue. Try to "RECORD" the number and types of moves you make. This chart may help: (Or devise a chart of your own.)

ORDER OF MOVES:	RED FREEBELS:						BLUE FREEBELS:					
	FROM:			TO:			FROM:			TO:		
	Red Square	Blue Square	OPEN	Red Square	Blue Square	OPEN	Blue Square	Red Square	OPEN	Blue Square	Red Square	OPEN
1st move												
2nd move												
3rd move												
4th move												
ETC.....												

Game 2: (If Game 1 is "too frustrating." Try placing only one Red Freebel and one Blue Freebel on Red Square 1 and Blue Square 1. Now following the rules, How many moves will it take to "interchange" their positions?

Game 3: Place 2 Red Freebels and 2 Blue Freebels. Put them on Red Square 1 and 2 and Blue Squares 1 and 2. (Increase number of Freebles used till all 10 are in play.)

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THREE-IN-A-ROW

Ticktacktoe frames can be used to practice a variety of math skills--standard processes; odd or even; partner of 10; fractions; Roman numerals; greater, less, or equal to a number; fractional equivalents; time or money equivalents; and area. Examples of how games can be plotted on chalkboard:

(1) Add 4:

2	5	1
6	3	8
7	4	9

(2) Multiply 3:

2	5	7
6	4	9
3	1	8

(3) Give Roman Numerals:

5	6	10
9	4	13
20	8	27

(4) Area of these squares?:

9 in.	5 ft.	4 in.
1 yd.	12 in.	36 in.
3 ft.	11 in.	8 ft.

(5) How many quarters?:

$1	#$\frac{1}{2}$	#$\frac{1}{4}$
$2	75¢	5 dimes
$5	25¢	20 nickels

DIRECTIONS:

With two children at the chalkboard, the players take turns choosing a number in the frame and saying the answer aloud. If it is accepted as correct, the player circles the number. For instance, if a player gave the correct answer for 1 + 4, 3 + 4, and 7 + 4, in the first ticktacktoe, he scores a point for getting three correct in a row.

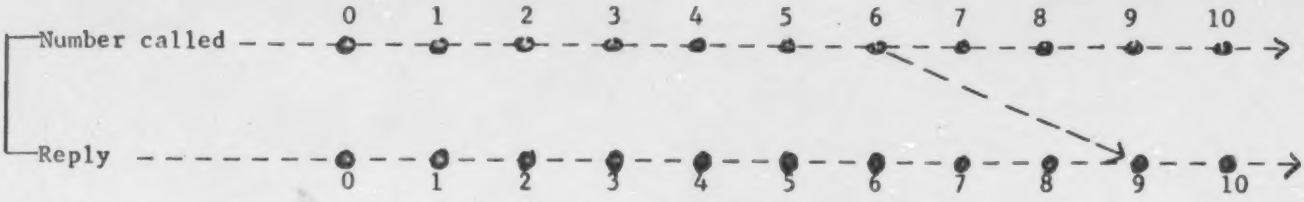
VARIATIONS:

Try to make up some variations of the "Three-in-a-row" with practices math skills not illustrated in examples 1-5 above.

Math Lab Activity--Patterns

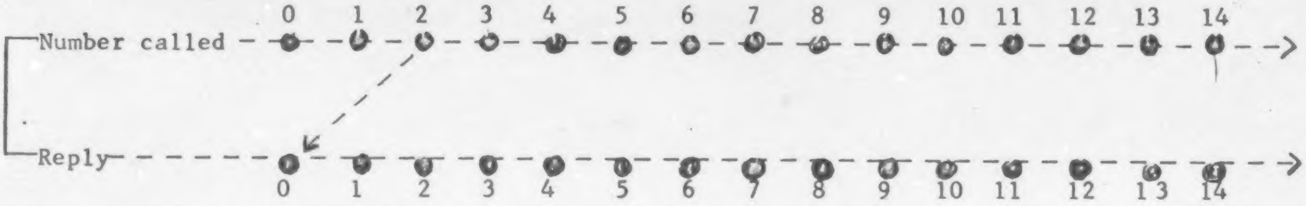
Game I:

	A	B	C	D	E	F	G	H	I
Number called	6	0	5		1		12	18	
Reply	9	3		10		6			32


Number called
0
1
2
3
4
5
6
7
8
9
10
Reply
0
1
2
3
4
5
6
7
8
9
10

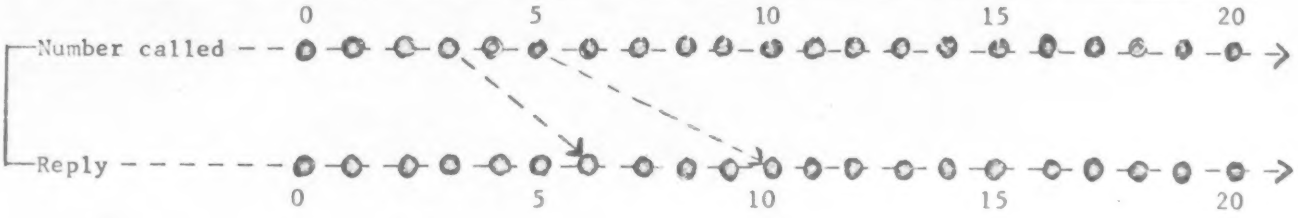
Game II:

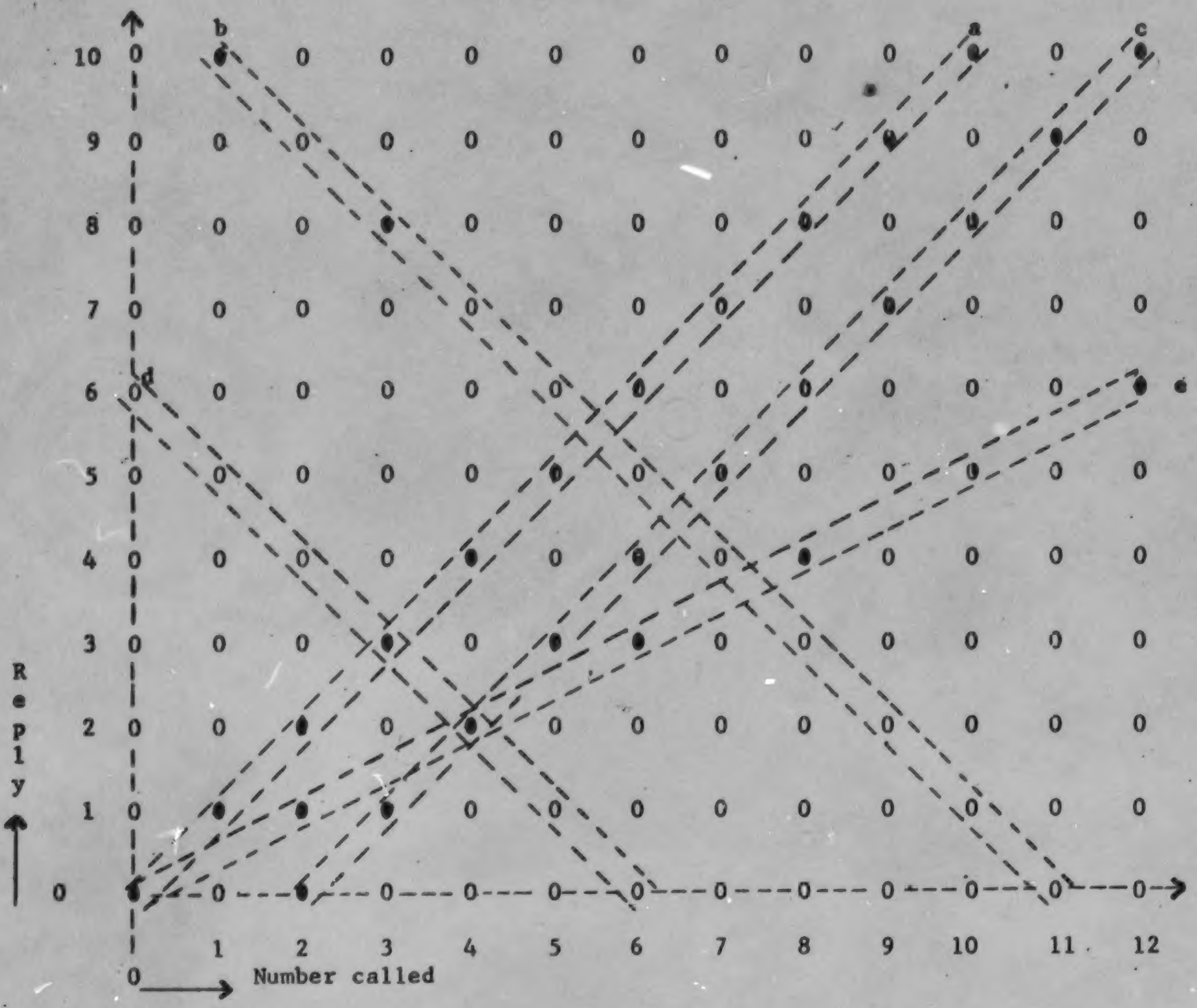
	J	K	L	M	N	O	P	Q	R
Number called	2	7	5			14		29	
Reply	0	5		6	10		18		38


Number called
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
Reply
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14

Game III:

	a	b	c	d	e	f	g	h	i
Number called	3	5	2		10			7	
Reply	6	10		8		0	20		18


0
5
10
15
20
Number called
Reply
0
5
10
15
20



Game III.

a

→	↑
0	0
3	3
1	
	10
2	

b

→	↑
1	10
3	8
11	
	4
	6

c

→	↑
2	0
9	7
4	
	6
5	

d

3	3
1	
	2
	0
2	

e

→	↑
10	5
0	
8	
	2
	1
6	
	6

Math Lab Activity -- Squares

Do You Know a Square When You See One?:

Here is a provocative activity designed to make students more aware of the care that accurate observation entails. Not many students will locate the full number of squares (40).

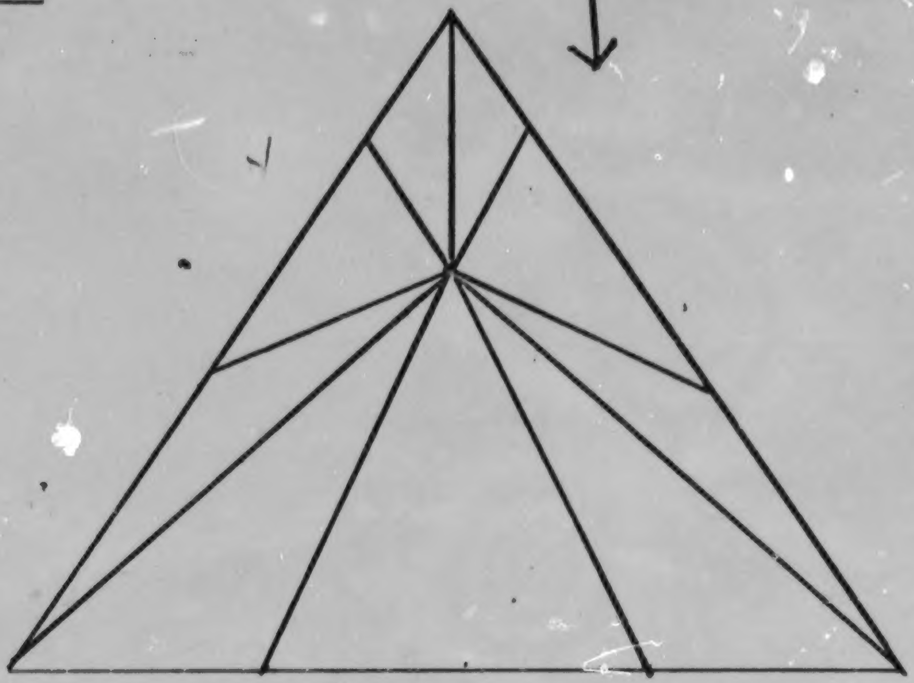
Give each student a dittoed copy of the diagram on page 1 of 2, Figure 1, so that he can work with it individually to determine his answer. Then give each person a slip of paper on which to record his answer. Names need not be attached, for the purpose is merely to demonstrate the wide range of answers coming from one group. Tally the responses quickly on the board.

Probably at least one student will have the answer of 40, so you can point out that answer and give the students more time to study the diagram. An ideal way to present the solution to this problem is on an overhead transparency that all can view at once as students take turns pointing out sets of squares. The drawing might also be quickly reproduced by a student on the chalkboard.

Work together in discussing the solution as students use their individual worksheets to identify the squares. They can mark different squares with patterned designs or outline squares with colored crayons or felt pens. Just to assist you in locating the squares, here is a list of those we've located (perhaps alert students will find more):

1	the whole square
16	divide the big square in 16ths
4	divide the big square in 4ths
4	squares composed of 9/16ths
2	small squares (divided in 4ths)
8	above squares, 4 each
1	central group of 4/16ths
4	4/16ths in center of each side

Encourage students to invent their own diagrams using squares in various ways as well as other geometric shapes. How many triangles, for example, can be found in this diagram?



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Math Lab Activity

TASK:______________________ NAME:______________________

BY-YOURSELF TASK	GROUP TASK
What you did:	What Group did:

Math Lab Activity -- Squares

By Yourself Task:

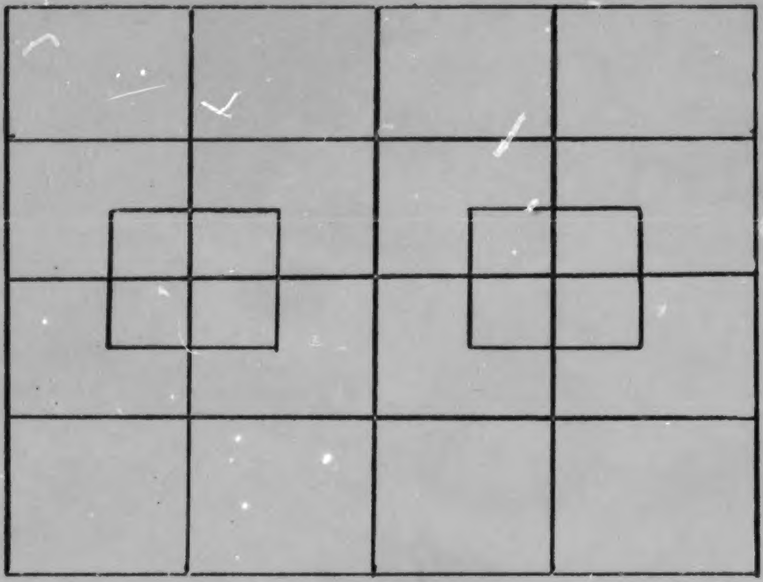
1. Determine how many squares there are in Figure 1. As you identify the squares, Write how you set out to determine the number of squares and the total number of squares that each attempt generated. (Use attached sheet.)
2. (When someone thinks he or she has located ALL the squares, say "stop" to other members of your group and at that time everyone in the group should record how you determined the number of squares and the total squares you found.)
3. Now move into Group Task.

Group Task:

1. Compare group members' methods of finding out how many squares and the totals you each found.
2. Answer the following questions:
   - a. Was YOUR method as accurate and productive as someone else's? Why? or Why not?
   - b. How could you have organized your method better to produce more accurate and speedy results?
   - c. What kind of thinking is involved in such problems?

(NOTE: See the Key for Math Lab Activity -- Squares for One Analysts. Try some others such as the Triangle Diagram shown in the Key.)

Figure 1:



Math Lab Activity

TASK:____________________ NAME:____________________

BY-YOURSELF TASK	GROUP TASK
What you did:	What Group did:

Math Lab Activity -- Assumptions

By Yourself Task:

1. Study the following problem:

$$\begin{array}{rcc} & a & b \\ + & r & m \\ \hline & o & b \end{array}$$

2. Try to find which of the following is true?
   - A. $a + b = m$
   - B. $a + m = b$
   - C. $a + m = r$
   - D. $a + m = a$
3. As you try to determine which of the above is true, WRITE the following:
   1. A list of each assumption to make or attempts you made, IN THE ORDER YOU MADE THEM. (Use attached sheet.)
   2. What discovery or what bit of information opened the door to the true response?
4. When someone thinks he or she has found the true response, say "stop" to other group members, and move into group activity.

Group Task:

1. Answer the following questions:
   - A. How many assumptions did each person list?
   - B. Were there any patterns to the order in which assumptions were made?
   - C. Can the group identify some strategy for attacking such problems?

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Math Lab Activity

TASK: ______________________

NAME: ______________________

BY-YOURSELF TASK	GROUP TASK
What you did:	What Group did:

CLASSIFICATION

We spend much of our time classifying the world. Pupils learn all sorts of classification schemes in school--from alphabetizing to learning the names of various plants and animals. They compare terms like big and little, and they test tastes like sweet and sour. In a surprisingly short time children master such complex tasks as finding a specific house among the hundreds in their neighbourhood. They can master such tasks because the classification schemes we use (such as street and house numbering systems) are often relatively simple even though they encompass large numbers of cases.

In addition to learning specific classification schemes commonly used in our culture, children also need to explore the basic nature of classification. They need to see that classification is essentially a useful, though sometimes arbitrary, means of grouping objects for easier identification. Objects are usually classified on the basis of several observable properties and conditions.

Telephone Book:

One way to emphasize the important point that any given set of objects can be classified in a number of ways is to re-classify a page from the telephone book. The telephone people classify people alphabetically, and this system seems to work quite well. All the more reason your pupils might leap to the challenge of discovering other systems.

Select a page from the book and transfer it to an overhead transparency (or you might be able to get a dozen or so copies of last year's book from the phone company). Your pupils' first task will be to create the first stage of your classification system -- to divide all the names into two (or more) subsets on the basis of an observable property.

What possibilities exist? Literally dozens. People living east and people living west (or north and south) of some major street in town, all people in each of the telephone exchanges used in your area, residences and businesses, phones owned by men and phones owned by women (Illustration 1).

Illustration 1: Divide the map of your city.

CLASSIFICATION

But why use such obvious classification schemes? Classify people by the number of letters in their last name, by the sum of the digits in their addresses, by the national origin of their names, by whether the last letter in their names comes in the first or second half of the alphabet.

Once your pupils get into the spirit of the task, they will rise to the challenge of discovering further divisions that will finally reduce each name on the page to a set of which it is the only member.

Discuss the systems your pupils develop and compare them with the system used in the phone book. What advantages and disadvantages are evident in each? (Don't think that an alphabetical classification is without its problems. Have you ever known an address but not the name, the first name but not the last?) Your pupils should discover that a classification scheme will remain in common use only if it is the most useful scheme that people can devise.

OTHER CLASSIFICATIONS:

Try the activities suggested below. Each will help you and your pupils discover a bit more about classification, and each should suggest further explorations you might make.

Ask your pupils to classify the words in the week's spelling list. For example, words might be grouped into various classification stages by whether or not they contained double letters (moon, seed, etc.) by the number of vowels they have, by the number of little words that can be made out of the spelling word and so on. When your pupils have completed their schemes, ask them to explain the schemes to each other. They will be surprised at the variety of criteria they used to group the same set of words.

Place a different object in each of a dozen sacks. Ask pupils to place a hand in each sack and to classify the 12 objects by feel alone, continuing to group the items until each one is in a class by itself. Compare classification schemes the pupils devise.

Classify the articles on the front page of your local paper using various criteria: good and bad news; local, national, and international news, the number of names used in an article; whether or not the article is continued on another page, headline size, and so on. After you've developed a scheme that classifies each article in a separate class, see if you can use the same scheme on front page stories of succeeding days.

Bring a sack of unshelled peanuts to class. Can a group of pupils classify the entire sack so that each peanut ends up in a separate class? Initial categories will probably be based on the number of peanuts in the shell, whether or not the shell has a crack in it, color variations, and other more obvious aspects. In time, though, the group will be challenged to discover more and more sophisticated measuring tools: soda straw balances, holes in cards to measure diameter, water displacement measurements, etc. It's quite a challenge, but it can be done.

CLASSIFICATION

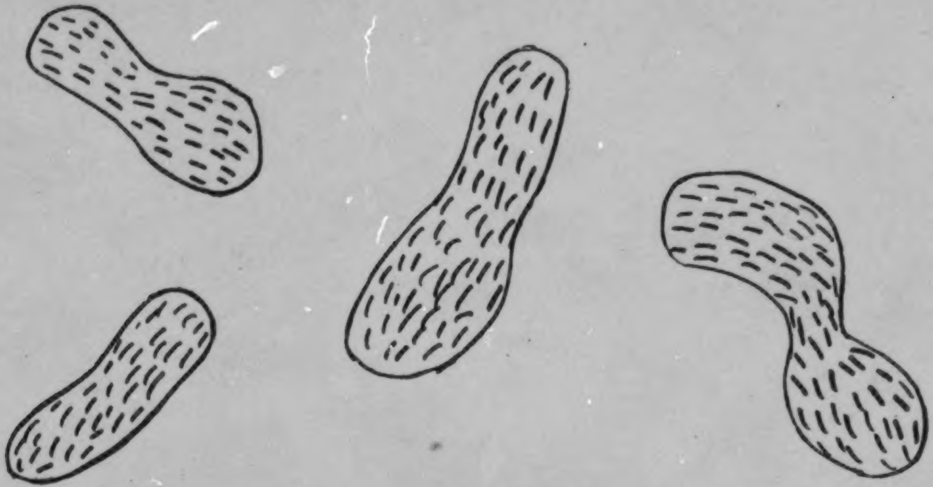


Illustration 2:

Ask each pupil to select an animal, but not to divulge it. Select one pupil as "it". Then announce various categories and ask pupils who fit the category to stand in a stated spot. All pupils who are not in "its" category are out of the game. Follow "it" from category to category until someone guesses what animal he is. For example, a sequence of categories "it" is in might be: zoo animal/four legs/spots/long neck. By this time someone will probably guess giraffe.

Call four pupils to the front of the room. Say, "I called these pupils to the front because they all have something in common that you can see. What is it?" Your pupils might note several properties or conditions common to all: blond hair, brown shoes, two ears, etc.

How many different ways can your pupils think of to classify all the pupils in your rooms so that each is in a unique class? Can they classify all the arithmetic books so each is in a separate class? How would they classify all the desks in the room? What about their hats and caps? Their hair?

Classification is an important thing to understand because classification decisions stay with us throughout life. What gets done and what is ignored? What gets done right now and what later? Which bills get paid in full and which in part? With whom do we associate professionally, socially?

FLOW CHARTING

Flow Charting--An Exercise in Logic:

Flowcharting is one of the preliminary logical steps taken by a computer programmer to designate the specific directions or necessary "yes-no" decisions that must be taken in defining a given behavior. Flowcharting introduces the idea of binary choices, relating it to decision making. Each time a decision point is reached, alternative courses are given with a binary (yes-no) decision on each course of action. The idea of binary choice narrows decision making to key choices; secondly, programming the steps clearly sharpens the logic or definition of the steps involved.

Flowcharting is fun and it allows the student to clarify the procedures in solving a story problem or refining computing processes. What are the steps in performing two-place addition? Multiplace multiplication? Long division? Flowcharting provides the student with the definition of the variables involved in the behavior in a way that he can follow step-by-step and verbalize. It is an aid to sharpen classroom thinking. Notice the basic symbols as you examine the flow chart on page 2 of 2.

◇ equals start ○ equals operation

⬡ equals decision.

Now begin flowcharting with your own classroom. Try some of the following topics:

1) How to get up in the morning
2) How to wash your face
3) How to brush your teeth
4) How to comb your hair
5) How to walk or ride to school
6) How to put on your shoes and socks
7) How to open the door
8) How to sharpen a pencil
9) How to check out a book from the library
10) How to play a piano or guitar
11) How to set your hair
12) How to use a television-newspaper log
13) How to add a two-digit number
14) How to multiply a three-digit number by a single digit number
15) How to divide a three-place number by a one-place number.

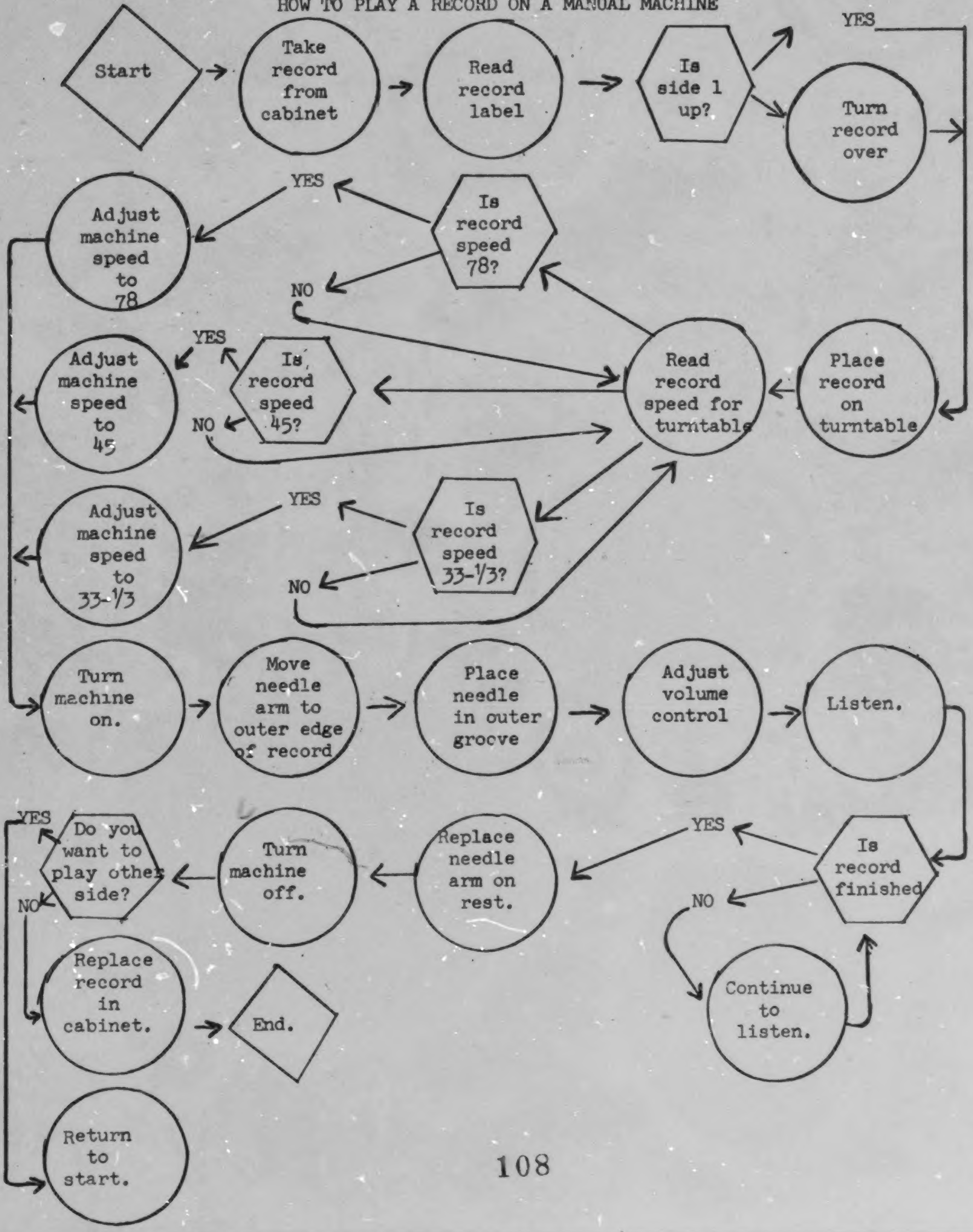
Flowcharting is one of the basic skills used in various demonstration school projects in teaching mathematics to disadvantaged children. For further information of two specific projects employing this technique write:

PROJECT R-3
Richard Cirigliano, Director
San Jose Unified School District
San Jose, California 95100

PERSONALIZED MATHEMATICS
Lewis A. Prilliman, Project Coordinator
Long Beach Unified School District
Long Beach, California 90801

FLOW CHART

HOW TO PLAY A RECORD ON A MANUAL MACHINE


Start
Take record from cabinet
Read record label
Is side 1 up?
YES
Turn record over
Place record on turntable
Read record speed for turntable
Is record speed 78?
YES
NO
Adjust machine speed to 78
Is record speed 45?
YES
NO
Adjust machine speed to 45
Is record speed 33-1/3?
YES
NO
Adjust machine speed to 33-1/3
Turn machine on.
Move needle arm to outer edge of record
Place needle in outer groove
Adjust volume control
Listen.
Is record finished
YES
NO
Continue to listen.
Replace needle arm on rest.
Turn machine off.
Do you want to play other side?
YES
NO
Replace record in cabinet.
End.
Return to start.

OBSERVATION

Any investigation of an object begins with those characteristics that are directly perceived by the senses. The investigator observes with all appropriate sense organs so that he can describe the object as accurately as possible.

The newer elementary science programs emphasize the development of observation. Use the activities below to help introduce your pupils to skills they need to observe accurately.

A Left Hand Laboratory:

It isn't necessary to purchase a lot of equipment to explore the nature of observation. You can begin with your pupils' left hands.

Divide your class into groups of four or five children. Ask each pupil to examine his left hand intently for several minutes, and then to write in precise terms all the observations he can make of his left hand (Illustration 1).

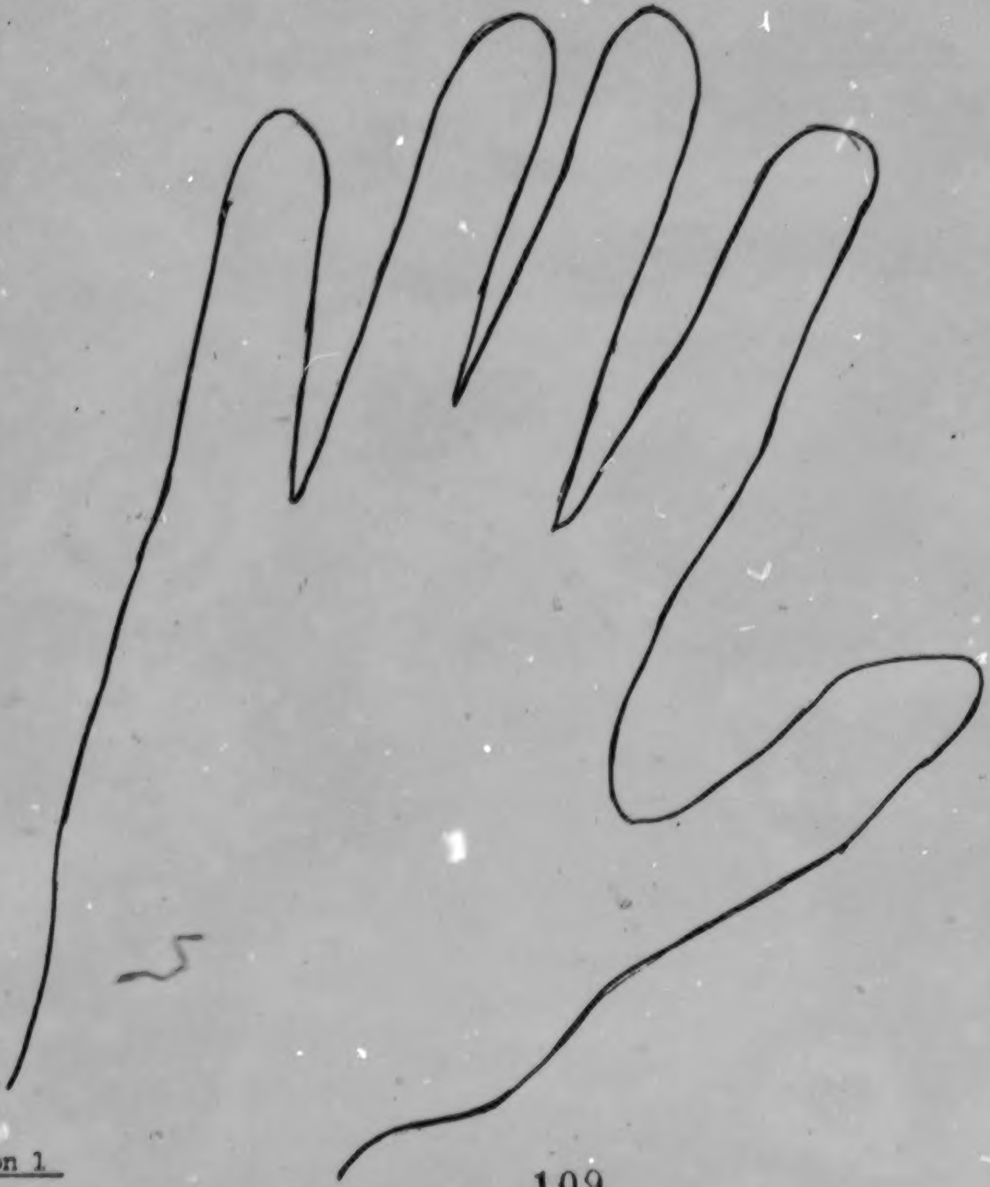


Illustration 1

OBSERVATION

Caution pupils to write only things they can observe directly, and not the interpretations they might make on the basis of their observations (inferences). For example, some students night observe long blue lines or ridges under the skin on the top side of their hand, but they only infer (and not observe) that these are veins.

Pupils should look for properties and conditions. A property is a trait or attribute of an object--the covering on my hand is flexible; the surface on my fingernails is smooth. A condition is a state of being--my hands are dirty; my fingernails have been cut recently.

Encourage them to ask questions such as: How many? How large? How long? What similarities and differences exist? Observation uses all senses, so children should smell, taste, feel, shake and listen, as well as look when they gather information on their hands. How might such tools as rules, pins, graph paper, calipers, and lengths of string help them make more accurate observations?

Perhaps some pupils will want to make their observations blindfolded, or with a glove on their left hand, or with their hand immersed in water.

After each pupil has written several observations, ask the members of each group to compare and discuss their lists, and to change or eliminate inaccurate observations. Were any observations made by everyone in the group? Were any observations true of only one person in the group?

Then bring all the groups together and discuss the observations the class made. It's probable that only two or three identical observations will have been made by every pupil in the class. Place the left hands of several pupils right next to each other to note the variation that exists. Arrange hands from shortest to longest, from darkest to lightest color, from smoothest to roughest skin.

No doubt about it, your pupils will treat their left hands with new respect from now on--and yours too--especially if you just happened to show up in school that day with a diamond ring weighing down your left hand!

Manufactured Variations:

People think that standardized manufactured items are identical. Careful observation will show differences, however.

Gather a dozen or so examples of each of several sets of such machine-made objects as ping-pong balls, paper clips, plastic spoons, pencils, etc. (Illustration 2). Divide your class into groups and give each group a different set of objects. Ask them to discover differences in their set of objects.

At first children won't be able to find differences, but it won't be long before they will figure out ways to discover the slight variations in weight, length, and color that exist. They will find nicks and cracks, marks and smudges--especially if they have access to a magnifying glass. Challenge them to discover a unique observation in each object in their collection of similar items.

After this experience, your pupils should be able to discover many differences in sets of such "non-manufactured" items as maple leaves, pine cones, flowers, twigs, apples, and carrots (Illustration3).

What 18 Hours Can Do:

To help your pupils become more aware of their classroom surroundings, ask them to make observations of their classroom the last thing one afternoon and the first thing the next morning, and to note any changes that occured overnight.

Shadows will have changed; a flower might have opened; the wastebasket will have been emptied; an open window might be closed; the temperature might have changed.

Go one step further. Give each pupil a special spot: a patch of floor, a chalkboard, a window pane, a desk top, a bulletin board. Ask pupils to observe overnight changes only in their assigned spaces.

OBSERVATION

Make changes in the room after the pupils leave each day. Are they able to catch more and more of the changes as time goes on, andespecially the less obvious ones?

Note the clothes your pupils wear from day to day. What differences can they observe on successive days? Do weather and clothes observations have anything in common?

Observation Objects:

Many common objects lend themselves well to imaginative observation activities. Start with these and see where they take you: a square of wax paper with a water drop on it; little piles of jello powder, Kool-Aid, chili powder, cornmeal, etc.; a potato chip or a cornflake; a little square of elm leaf or crepe paper; a piece of tapioca.

Observe objects that will change during the observation period: an ice cube; a birthday candle; a cube of jello; a sugar cube dropped in water; an ink drop dropped in a glass of warm water.

And, of course, the most changeable set of observations your pupils can make is an observation of you every fifteen minutes during the school day. Won't their lists make interesting reading after school?

"Now let's see. Is that an observation or an inference?"

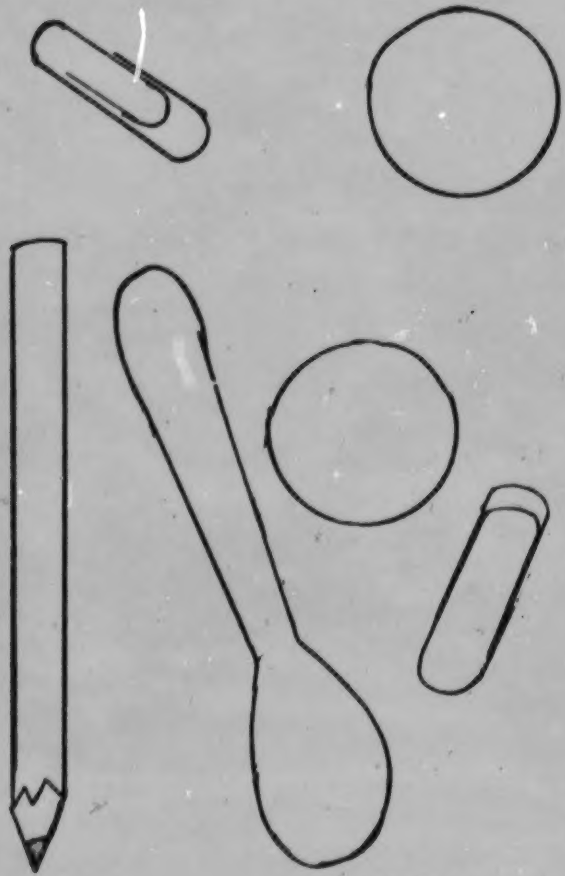


Illustration 2

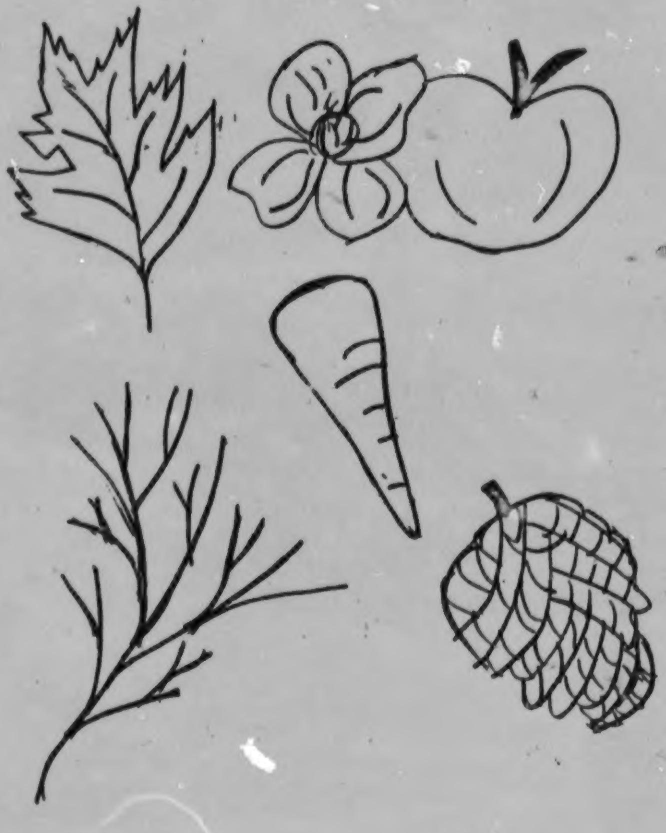


Illustration 3

STORY PROBLEMS

"Do I multiply or Divide?"
"I can't read this story problem."
"I can add on the practice page, but I can't do story problems."

Sound familiar? Perhaps you have the kind of child in your class who is at-level in achievement but falls down on story problems. One interviewer reports such a case. When asked how she did story problems, the student replied, "If there are lots of numbers, I add. If there are only two numbers with lots of parts, I subtract. But if there are just two numbers and one littler than the other, it is a hard problem. I divide to see if they come out even. If they don't, then I multiply to get the answer." As you can see, although this student may not understand the story problem, she does have an intelligent attack pattern. Try some of the following ideas instead of your next page of story problems. See if your student will not develop better attack strategies.

MATH HEADLINE STORIES:

Put two numbers on the chalkboard and have children make up a story and a problem involving the numbers. At first very ordinary problems are given: "If I had 5 marbles and someone gave me 7 more, how many would I have?" Note the headline: 5 + 7 = ? You can challenge children to be more creative at this point. After prompting, you might get this story: "I came home 5 minutes late and mother scolded me for 7 minutes. How much time did I miss watching my favorite TV cartoons if mother kept me from watching it for both the number of minutes I was late and the time she spent bawling me out?"

Of course, the advantage to headline stories is that they use the children's vocabulary and interests. Reading and motivation problems are overcome at once. Once you begin writing stories, develop the one basic rule: you must be able to solve any headline you write. As the teacher, you can challenge the student to make up headlines with difficult numbers or involving various operations. (See the sample headlines in Illustration 1.)

HOW MANY HEADLINES:

Give the children 2 numbers and allow them to choose any other number and as many operation signs as they wish. See how many headlines can be made with the two numbers. Develop a chain story based on the headlines.

REWRITE TEXTBOOK STORY PROBLEMS:

With this variation, first you read through the next page of story problems. You and the children will probably find these to be uninteresting. Challenge children to rewrite the story problems using exactly the same numbers and operations but building the story around a more interesting topic.

MATH HEADLINES FROM THE NEWSPAPER:

Use your local newspaper or a student current events weekly such as Scholastic or Weekly Reader as the source of story problems. Have the children select news items involving numbers. Develop math headlines and write story problems to be exchanged with your neighboring teacher's students.

STORY PROBLEMS

WRITE PROBLEMS WITHOUT NUMBERS:

In these examples, children solve your problems which are written to emphasize process without concentration on specific numbers. The following should get you started:

> If Bill had some money and spent part of it, how would you find how much he had left?
>
> Billy got some more toy cars for his birthday to add to his collection. How would you find out how many he has now?
>
> Sue was several inches shorter than Ruth last year. Now Ruth is the same height as Sue. How could you find out Sue's height now if you knew Ruth's height now and last year?

SOLVE GRANDPA'S PROBLEMS:

In this technique the teacher finds an old mathematics text, and using the language, and flavor of that period, has a pproblem-solving day. A good way to present a page from the old text is to photocopy it and make a Thermofax overhead projection slide. Together you and your class can attack the problems from the old text.

A similar procedure is to obtain old records such as the 1849 miner's bill of fare or a menu from an Old English Inn. Make up "Math Headlines" based on the old records. Perhaps children might write a page of problems from the old records that might have appeared in Grandpa's textbook.

WOULDN'T YOU LIKE TO SOLVE A HIGH SCHOOL ALGEBRA PROBLEM?:

By securing high school text, you can find relatively simple problems for the children to solve. Simply tell them that if they will replace the question marks in their headlines with x's they have an "algebraic equation" to solve. Now, can they brag! We're doing algebra. Of course, they are solving simple equations and we are not pursuing the algebraic content but have at our command a powerful motivating technique.

DEVELOP A CLASSROOM STORY-STARTER SHEET:

11	baseball game	7
23	birthdays	15
20	spelling test	3
5	book club	32

STORY PROBLEMS

From everyday events in your class, a chart, such as the one above, lists events that have happened with our class. The numbers are not necessarily listed with the event but are distributed along each side. Children choose an event and a number from each side to make up math headlines and accompanying story problems. In this case the events and corresponding numbers are:

Girls beat boys in the baseball game, score 7 to 5.

We had 3 birthdays this month: each child was age 11

There were 20 words on the spelling test and 15 children missed 5 words each.

This month we brought 23 selections from the 32 on the book club list.

From the events, which children search to identify, they develop story problems for others to solve such as:

If 15 children missed 5 words, how many words did they miss all together?

15 x 5 = ?

Children soon become much more ingenious in adding math events to the list. Perhaps you will allow them to write their problems, in their own handwriting, for direct duplication.

Remember the basic rule: You must be able to solve any math headline you make up.

HEADLINES

Illustration 1

$\boxed{2,343,888 + 3,272,203 = \qquad}$

Brown and Reagan ran for governor. Brown had 2,343,888 votes. Reagan had 3,272,203 votes. How many registered voters voted for governor?

PLAN B $\boxed{----+}\boxed{387}\boxed{= 28,143}$

15,871 people voted yes on Police Building Bonds. 28,143 people voted no on it. How many more did they need to tie up the vote?

STORY PROBLEMS

$\boxed{2,408,024 - 1,769,942 = \qquad}$

Governor Brown had 1,769,942 votes.
Ronald Reagan had 2,408,024 votes.
How many more votes did Ronald Reagan have than Governor Brown?

$\boxed{2,203,865 + ? = 5,737,779}$

Anderson and Finch ran for Lt. Gov. Anderson had 2,203,865 votes. Together they both had 5,737,779 votes. How many votes did Finch get?

$\boxed{730,749 - 1,086,332 = ?}$

Brown had 730,749, Reagan had 1,086,332 votes. How many votes less did Brown have then Reagan?

$\boxed{9,987,654,321 + 20,000 = ?}$

9,987,654,321 people voted for Reagan. Then 20,000 more people voted for Reagan. How many people voted for Reagan?

Math Lab Activity -- Graphs and Curves

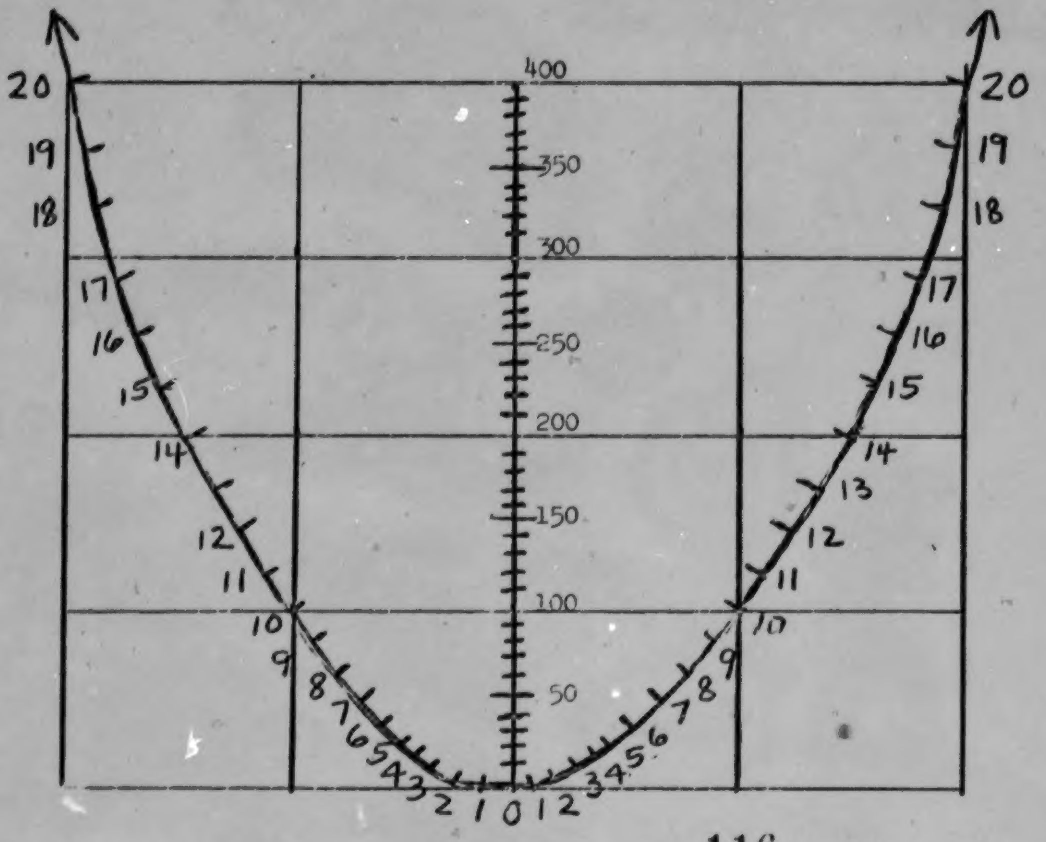
By Yourself Task:

1. (Take about 5 or so minutes.)
   Study Figure 4 and try to discover what you can by using the information given.
2. Identify what you did during each attempt you made to discover what the figure could be used for. List your attempts on the sheet under the column By-Yourself Task. (See attached sheet)
3. After about 5 minutes or until someone in the group calls "stop", move to group activity.

Team Task:

1. Compare your attempts and your discoveries orally within the group.
2. Then, try to respond to the following:
   a. Why did it work?
   b. What halped you most in making a discovery?
   c. Would it work if you changed the shape of the curve?
   d. Would it work if you changed the numerical scales on the graphs?
   e. What would happen to your discovery if you were to use angles rather than curves on the graph?

Figure 4:


400
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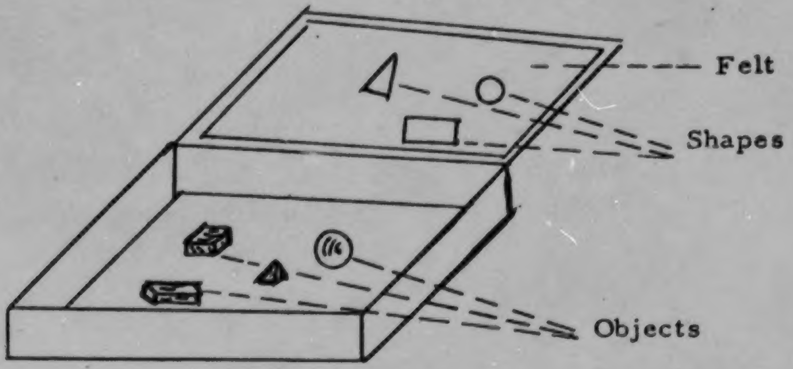
MATH LAB ACTIVITY--Cigar Box
Individualized Flannel Board

Learner manipulation of physical objects provides essential concrete experiences for developing concepts in math. Direct manipulation and exploring is important, BUT must be supplemented and at some time supplanted with representations of the concrete experiences in the form of pictorial and/or symbolic forms.

A manipulating device which lends itself to such representation tasks is the flannel-board. Learners can often manipulate materials which are placed on the classroom size flannel board, but they can often benefit more from making and using one designed for individual use. Such a flannel board can be made from a cigar box and sets of manipulative objects which will fit into the box. The inside of the cover has a piece of colored felt glued to it so that when the box is open the felt surface is ready to be used. A Cigar-Box Flannel Board can start as an Art project or as a parent-help project.

ILLUSTRATION:



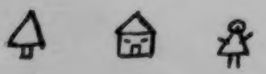
By including both manipulative objects and felt pieces in the Cigar-Box Flannel Board, it allows learners to explore concrete and semi-concrete experiences with number concepts.

Examples of manipulative objects which can be included:

- Small plastic toys--animals, miniature cars, people, etc.
- Discs--checkers, plastic pieces, bottle caps, etc.
- Small blocks--wooden
- Tongue depressors or popsicle sticks
- Paper clips

Examples of felt pieces:

Shapes: Triangles, squares, rectangles, cubes, semi-circles, etc.
(These could profitably be of different sizes and colors.)

Cut-outs of trees, people, houses: 

Numerals: 1 - 2, etc.

Bars----of different lengths and different colors.

MATH LAB KITS

What is a Math Lab Kit?

A Math Lab Kit is a particular manipulative aid or set of learning materials which may contribute to multiple uses by learners in many areas of math concepts. These aids and materials are not commonly viewed as being as versatile as they really are. The Intern is encouraged to experiment with these and other materials which might be developed into kits. These will be particularly useful by learners in laboratory explorations. Activity Cards and Guidesheets can be built around these kits by identifying learning objectives and then writing a series of Cards or Guidesheets in terms of those objectives.

Purpose of the Kits:

To provide a source of many learning activities for math laboratory explorations by learners. They can not only use Activity Cards and Guidesheets with the Kits but should be encouraged to invent new and unique ways to use the Kits to explore any math problems, ideas, or concepts.

Use of the Kits:

The Intern should initially explore the Kits himself in order to become familiar with the aid and some methods of using the aid with learners. The Kits are only beginnings; Interns should explore many other variations for their use. Many of the Kits can serve diagnostic uses; that is, for formative evaluation as well as summative evaluation which occurs at the termination of a learner's study.

Individual learners can utilize the Kits but learning is much more enhanced when 2 or 3 learners explore Kits together.

LIST OF MATH LAB KITS

1. Inch Blocks
2. Papy's Mini Computer
3. Geo-Boards
4. Number Lines
5. Cuisenaire Rods
6. Attribute Blocks
7. Place Value Devices
8. Fraction Kit
9. Games
10. Array Boards
11. Venn Rings

MATH LAB ACTIVITY KIT #1: INCH BLOCKS

What are INCH BLOCKS?

Uniform blocks which are 1" cubes, made of hardwood (preferrably, but not limited to hardwood or wood), and may be left unpainted, painted or finished in a clear finish. The blocks are made uniform in order to satisfy some of the experiences for which they are especially useful.

Availability of INCH BLOCKS:

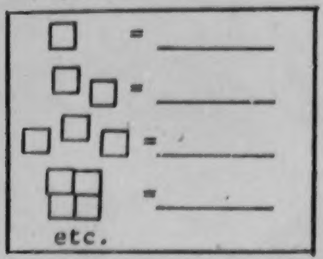
Many school supply houses and publishers of math aids sell such blocks in lots of 100 or more and the costs are quite reasonable.

The blocks can be cut if a person has access to a fine-toothed table saw but this requires some careful selection of material. Hardwoods are best but pine or similar ope-grain woods will suffice. If a fine-tooth saw is used, a minimum of sanding is necessary. If the blocks are painted, care should be taken to use non-lead-based paints or stains.

Uses of INCH BLOCKS:

1. COUNTING EXPERIENCES:
   Blocks may be manipulated in relation to the following counting expereiences:

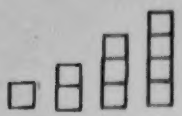
   a. 1:1 correspondence:
   Learners can be asked to select a certain number of blocks from a box of blocks and to count them orally as they are placed on a table. This process can be reversed by having the child match the blocks with square shapes made on a paper:

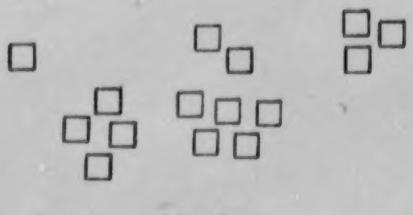


   As the blocks are matched the learner writes the numeral which corresponds.

   b. Comparison of numberness and order:

   Learners can be asked to count out certain number of blocks and put them in separate piles. When several piles are in front of him, the learner could be asked to stack the blocks in each pile and then arrange them from the fewest number of blocks in a stack (possibly 1) to the most number of blocks in a stack:


(ordered stack)


(random piles)

Use of INCH BLOCKS: (Cont.)

1. b. (Cont.)

Then he can be asked to "comment about" the stacks. (If directives to learners are left open--whether these are given by the teacher or are directions on a LAB ACTIVITY CARD--then the learner will have an opportunity to reveal what patterns he is discovering:

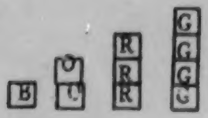
"it's a stair step"
"each pile has one more in it"
"there are 1, 2, 3, 4, . . .blocks in these stacks"
"there is one more in this stack than that stack"
"each stack to the left of another stack has one less than the stack and and each stack to the right of a stack has one more than the stack."
"etc."

c. Ordinal counting:

If colored blocks are used, learners can easily be led to detect more about the "relative position" of a number of blocks. For instance, the learner could be led to make a stair case of blocks by taking the following out of a box of blocks:

"1st take 1 BLUE BLOCK"
"2nd take 2 ORANGE BLOCKS"
"3rd take 3 RED BLOCKS"
"4th take 4 GREEN BLOCKS"
"Etc." (until several stacks are taken out.)

The display of blocks might look like this:



etc.

Now, the learner could be asked to complete the following kind of information:

"The 3rd stack of blocks are _______________(color)"
"There are ___________blocks in the 3rd stack"
(number)
"The 5th stack of blocks are _____________(color)"
"There are ___________blocks in the 5th stack."
(number)
"There is/are _______ less or more blocks in the 3rd stack than the 5th stack."
(number)
(Learner would indicate less or more by underline or circle.)

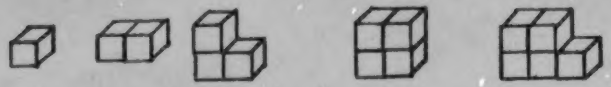
(Some learners could be introduced to the "greater than" > and "less than" < symbols and asked to fill in such items as this:
"The 6th stack is ______ > or < the 3rd stack." (Again circling the > or < symbols.)

Some "oral games" can be played by small groups of learners with these ordered stacks.

One player who is "it" could say "I have two stacks in mind. One is 3 more than the other. What are they." OR "How many of the orange stack do I need to make the green stack?" (This gets into work with multiples which builds experiences for generalizing about multiplication and division.)

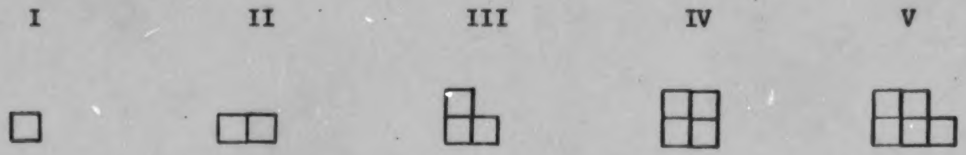
Use of INCH BLOCKS: (Cont.)

2. PRIMES and COMPOSITES:
To help learners differentiate between prime and composite numbers, which lays a groundwork for factoring, the blocks can be "BOXED" to reveal their divisibility. For instance, blocks to represent numbers could be arranged as follows:



Etc. . .

The BOXING is done in ROWS so that it is easily observed how the various BOXES may be divided.

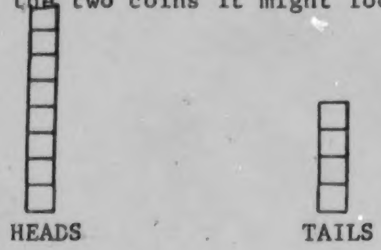


Etc. . .

For instance, Box III is made up of an "uneven" row or arrangement--☐ and ☐☐ , BUT BOX IV is made up of an "even" row or arrangement-- ☐☐ and ☐☐ . Those that make "even" rows are "PRIME," those that make "uneven" rows are "COMPOSITE." The Learners might be presented a series of LAB CARDS which have them do the BOX TEST on numbers from 2 to 15 or even higher numbers. This experience is close to that provided for in the SIEVE OF ERATOSTHENES method for finding PRIMES and COMPOSITES.

3. USING BLOCKS TO RECORD DATA DURING EXPERIMENTS:
Blocks that are uniform in shape such as inch blocks, can serve a VISUAL FORM of verification of data collected during various experimenting. For instance, if 2 coins (say pennies) are flipped 10 times, how many heads and how many tails come up? As the coins are flipped (simultaneously) someone can place blocks on the table or chalk board to "show the results:"

After 6 tosses of the two coins it might look like this:



At this point in the 10 tosses, it is helpful to ask the learner, "What do you notice?" He may respond that. . .

"There were more heads than tails."
"There were 8 heads and 4 tails."
"Four more heads came up than tails."
"Etc..."

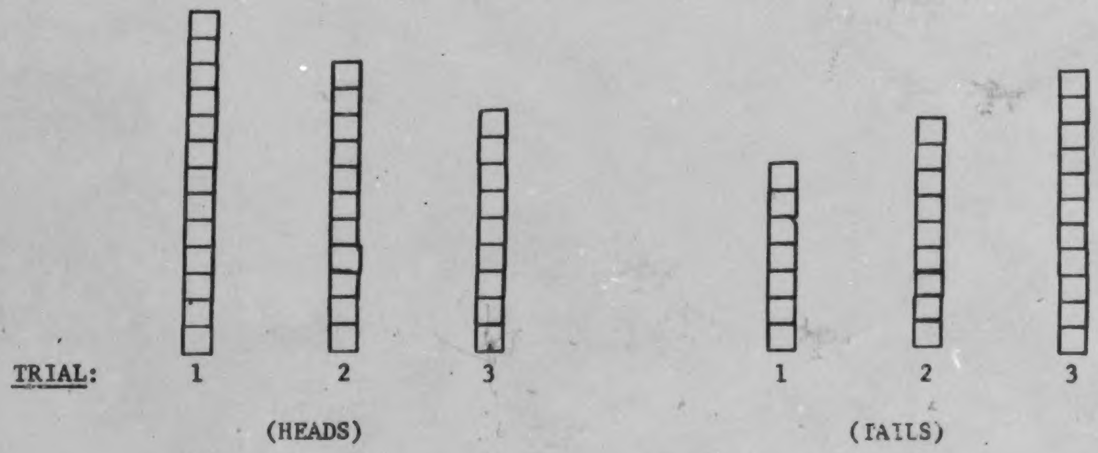
(Some "sharpie" might even state that "twice as many heads as tails have come up." OR "half as many tails as heads came up.")

Use Of INCH BLOCKS: (Cont.)

3. (Cont.)

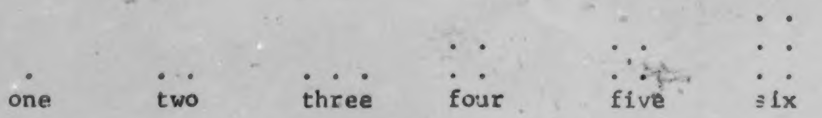
You could even probe the extent to which the data suggests final results of 10 tosses by asking:
"Do you think the tails will catch up?" OR
"After 10 tosses, will you still have twice as many heads as tails?"
(It is interesting to note that the teacher's role at this point is to avoid being an answer-seeker and emphasize more that the learner interpret, predict, and test about his own data!)

After completing the 10 tosses, the learner could be encouraged to repeat the 10 tosses 2 or more times and keep the results of all 3 attempts to record data for 10 tosses. Then, cause the learner to compare, contrast, and conclude or predict for further testing! This will keep inquiry open and allow the learner to follow ideas which he has developed. The learner might also be encouraged to "make a picture" of what his blocks show. This will introduce him to a HISTOGRAM or GRAPH of his data:



Results of 3 trials to toss 2 coins 10 times

Another interesting use of the blocks to record data is to display "the number of times each of the numbers (dots) on a DIE (one DICE)--1, 2, 3, 4, 5, 6--come up when the DIE is tossed 50 times." (or some other number of times) These data can be displayed along the chalk board (to allow written notes to accompany the stacks) and again summarized on a chart:



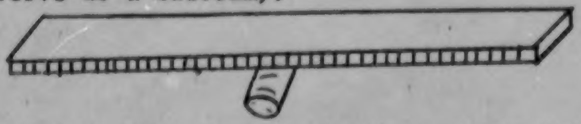
RESULTS OF 50 TOSSES OF A DIE (ONE DICE)

Use of INCH BLOCKS: (Cont.)

3. (Cont.)

Of course, one excellent chart that migh be made is the line graph of the above histogram. This will tend to show the "smooth line" of the results which tends to indicate the probability of an equal number of each number on the die coming up. Learners might speculate on when (if ever) they are able to display that all numbers have come up an equal number of times. (It can happen, can't it?)

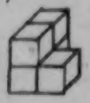
4. BALANCING BLOCKS:
Uniform inch blocks, especially if their weights are fairly accurate from one block to another, can be valuable in "balancing activities." A small board (2 or so feet long) can serve as a balance board and a small round block (cylinder) can serve as a fulcrum).



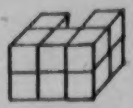
If the board is marked off in equal divisions, some interesting experiments can be tried. Placement of 2 inch-blocks at one point on the left of the board might be "balanced" with 1 inch-block at another point on the right of the board. (Some interesting activities are outlined in a HANDOUT in Module CMM:006:05(ISU) entitled "MATH LAB ACTIVITY: BALANCE BLOCKS.")

5. ATTRIBUTE GAMES WITH INCH BLOCKS:
Though limited by the fact of using one shape of block (cube), the "attribute work" can be quite interesting and stimulating. Many of the attribute games can be placed on LAB ACTIVITY CARDS.

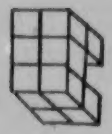
Provide the learner with a sketch (scaled drawing) of a shape such as the following and ask him to "arrange inch blocks" to show what the drawing shows.

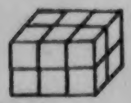


In this case the "back-lower block" is hardly visable. Increase the difficulty of the sketches as follows:



OR

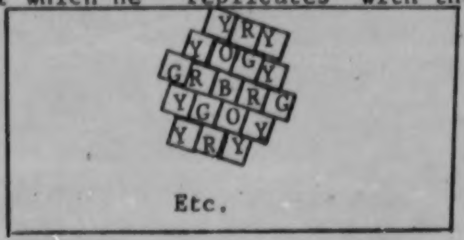




Etc.

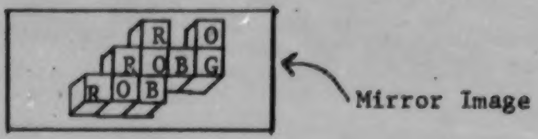
Use of INCH BLOCKS: (Cont.)

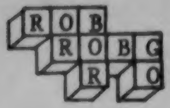
6. PATTERN WORK WITH INCH BLOCKS:
Many symetrical and unbalanced patterns may be tried with multi-colored inch blocks. The learner may "create" his own designs or be "given" a design on a piece of paper which he "replicates" with the blocks:



B-Blue
R-Red
O-Orange
G-Green
Y-Yellow

A variation of this design work can be gained by the use of a "small mirror." Make a small design of blocks, such as the following, then place the mirror to one side of the design to gain it's "reflection" and then try to make a similar "complete" design by including the design and it's reflection:





7. TRY TO DESIGN, CREATE, AND LOOK FOR MANY OTHER WAYS TO utilize inch blocks. Try to design LAB ACTIVITY CARDS for learners to use independently of teacher.

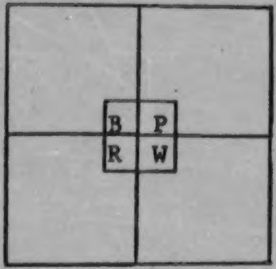
MATH LAB ACTIVITY KIT #2:
PAPY'S MINICOMPUTER

What is the Mini-Computer?

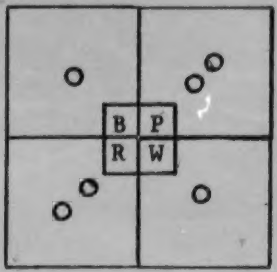
The Papy Minicomputer is a development that is related to the Cuisenaire Rods and use of the computer should be preceeded by rather complete familiarity with at least the following Cuisenaire Rods and equivalent values:

White Rod = 1 unit
Red Rod = 2 units (2 whites)
Purple rod= 4 units (4 whites OR 2 reds)
Brown Rod = 8 units (8 whites, or 2 purples, or 4 reds.)

The mini-computer in its simplest form is a sheet of paper (or other appropriate surface) marked as follows:



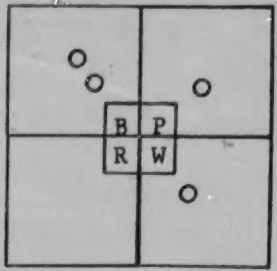
The above arrangement allows the learner to use the equivalent values of W, R, P, and B to work out problems involving simplifying numbers, addition, subtraction, and other operations on numbers. As an example, what is the number represented by the marks (dots, chips, or other markers) shown on the computer below:



(1 eight, 2 fours, 2 twos, and 1 one OR 8 + (4 + 4) + (2 + 2) + 1 = 21.)

Could this be "simplified" in any way on the computer?

How about this?

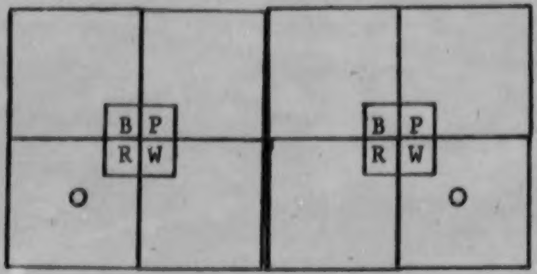


What is the Mini-Computer? (Cont.)

The 2 fours could be "combined" to make another 8 and the 2 twos "combined" to make another 4, thus we now have (8 + 8) + 4 + 1 = 21.

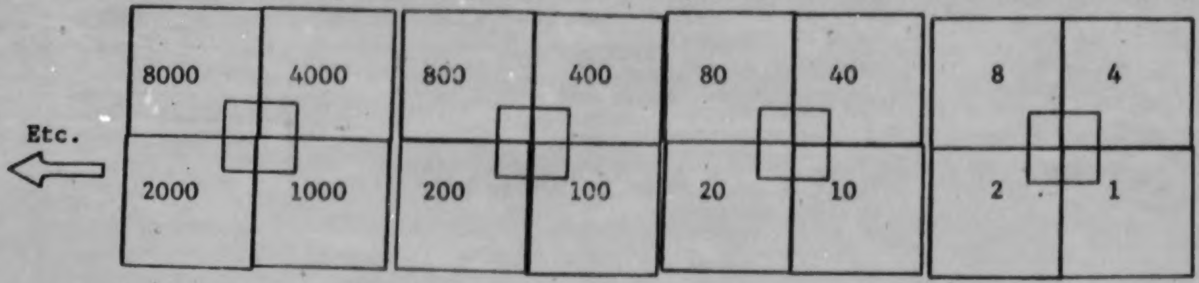
Can the 21 be further simplified?

The answer is YES, IF the computer is expanded:



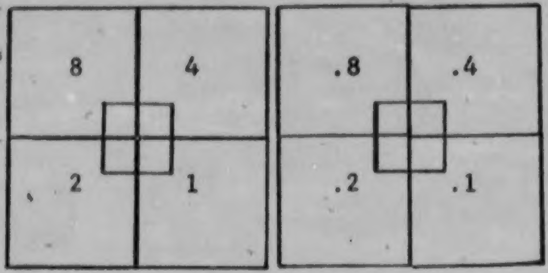
By adding another "computer storage" to the left of the original which would store 8's, 4's, 2's, and 1's. The New Storage will store 10's, 20's, 40's, and 80's; thus the above shows 20 + 1.

The mini-computer is thus a way to display "expanded" numbers and "simplified" numbers. Of course, the computer can be expanded as far as needed. For example:

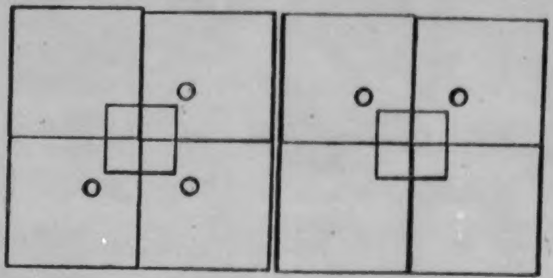


It can also be expanded to the right to show the expansion of the number system into rational numbers. Thus we have:

FIGURE A.



What would this number be if we used the Fig. A computer?



(4 + 2 + 1 + .8 + .4 = 7 + 12 tenths =

7 + 1.2 = 8.2.)

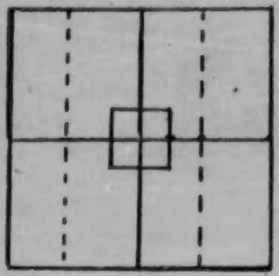
What is the Mini-Computer? (Cont.)

Many activities and operations are possible utilizing this interesting device. REFER to the following Handouts for additional ideas and to apply the Mini-Computer:

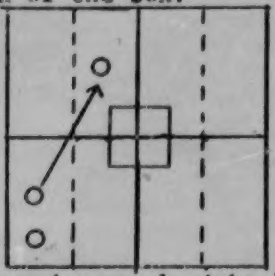
1. Arsdel and Lasky: "A Two-Dimensional Abacus--The Papy Mini-Computer" (This Handout is attached to Module CMM:006:07(ISU) so request that module any time you wish. It is an excellent "Individualization tool.")

2. "The Minicomputer" CMM:006:05(ISU)

3. A tablet of "minicomputer worksheets."
These are especially useful if 3 to 6 of them are laminated. In that way they will last longer as they are handled. The double column for each box as shown on the worksheet serves a special purpose:



As the markers are being placed they can be located in the left-hand portion of the box. Then as the learner "operates on those markers," they can be shifted to the right-hand portion of the box:



(2 fours become 1 eight.)

In this way, the learner can keep track of the "value" of numbers he is working with and those still remaining.

Variations of Mini-Computer:

The mini-computer device may be constructed of any of the following materials:

1. The mini-computer worksheets that have been laminated. Most any marker (chips, discs, or heavy cardboard cutouts) is usable on them.

2. A metal surface (galvanized sheet metal) which has been marked off with paint or tape. Magnetic markers serve well and allows a stand-up display of the mini-computer.

3. A wood or heavy cardboard model can serve as a more permanent surface, especially if it is finished with a hard surfaced paint.

BROWN	PURPLE
RED	WHITE

The Mini-Computer

WHAT IS IT?

The Mini-Computer is a device which employs concepts developed with the study of Cuisenaire Rods but which utilizes aspects of a place value device. The colors--white, red, purple, and brown--refer to the Cuisenaire Rods with those same colors. The values of the rods are:

WHITE = 1 unit
RED = 2 units (OR 2 WHITES)
PURPLE= 4 units (OR 4 WHITES OR 2 REDS)
BROWN = 8 units (OR 8 WHITES, OR 4 REDS, OR 2 PURPLES)

Another way to represent the values of the 4 positions on the mini-computer is:

$$8 = (4 + 4) = (2 + 2 + 2 + 2) = (1 + 1 + 1 + 1 + 1 + 1 + 1 + 1).$$

The mini-computer itself is a sheet of paper with the 4 colors or simply a divided sheet of paper. Several sheets of mini-computers are needed to operate the computer, however. This will be evident later on.

GROUND RULES FOR OPERATING THE MINI-COMPUTER:

1. Keep in mind the ratios of value of the 4 squares--WHITE = 1, RED - 2, PURPLE = 4, and BROWN = 8.

B = 8	P= 4
R = 2	W= 1

2. COUNTERS OR MARKERS are placed on the mini-computer to "display" a number. For instance, the following shows 7:

	o
o	o

(4 + 2 + 1 = 7)

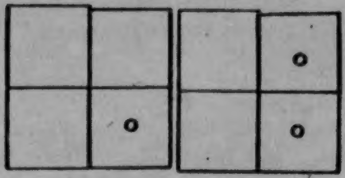
3. The mini-computer, like all computers, can "expand its storage," thus to show 15 on the MINI-COMPUTER, we "expand" by adding another sheet but with the ratios of 10, 20, 40, 80:

80	40	8	4
20	10	2	1

GROUND RULES FOR OPERATING THE MINI-COMPUTER: (Cont.)

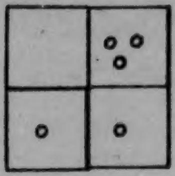
3. (Cont.)

Now we can show 15 as:

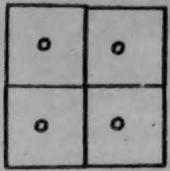


(10 + 4 + 1)

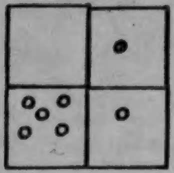
(NOTE: We could show 15 in a number of ways, but the above example is the "simplest form.")



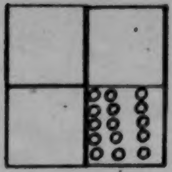
= 15



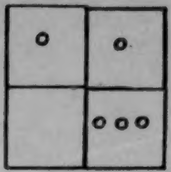
= 15



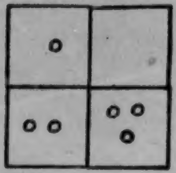
= 15



= 15



= 15



= 15

To represent numbers on the Mini-Computer we always seek to simplify them.

4. The Mini-Computer may "expand" in both directions from the basic computer:

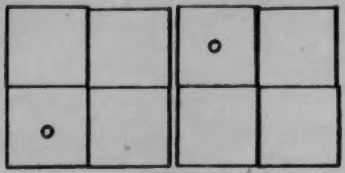
8000	4000	800	400	80	40	8	4	.8	.4
2000	1000	200	100	20	10	2	1	.2	.1

(Basic Computer)

5. When computing on the Mini-computer we can use counters, markers, (even marks) that are alike or different to show operations on numbers:

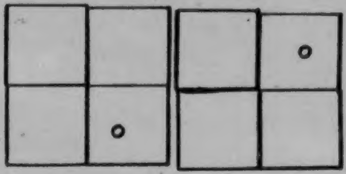
a. ADDITION OPERATION:

28 + 14 = 42



28

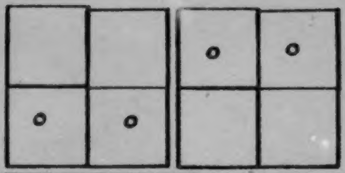
+



14

When these numbers are "combined" on one mini-computer, they "simplify" as follows:

STEP ONE:

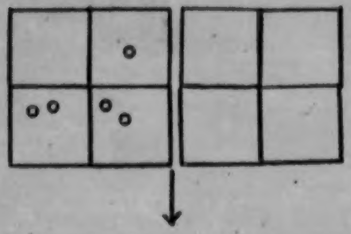


Put both 28 and 14 on same mini-computer.

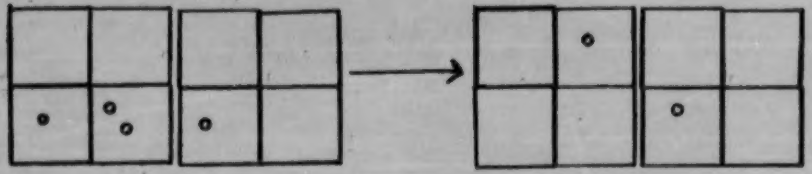
GROUND RULES FOR OPERATING THE MINI-COMPUTER: (Cont.)

5. (Cont.)

a. STEP TWO:



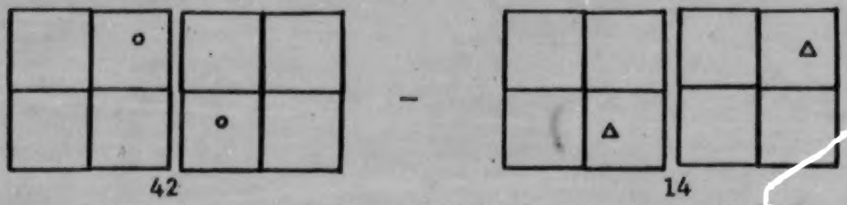
Now, to "combine" 28 and 14 to "simplify" our display in the computer, we must regroup our 4 into 2 twos, then regroup the 8 and 1 two into 1 ten which we place with the other 10, and lastly regroup our 1 twenty and 2 tens into 1 forty; therefore, our final "display" shows 40 + 2 or 42.



The above basically explains the "addition operation" but subtraction, multiplication, and division involve different procedures on the Mini-computer.

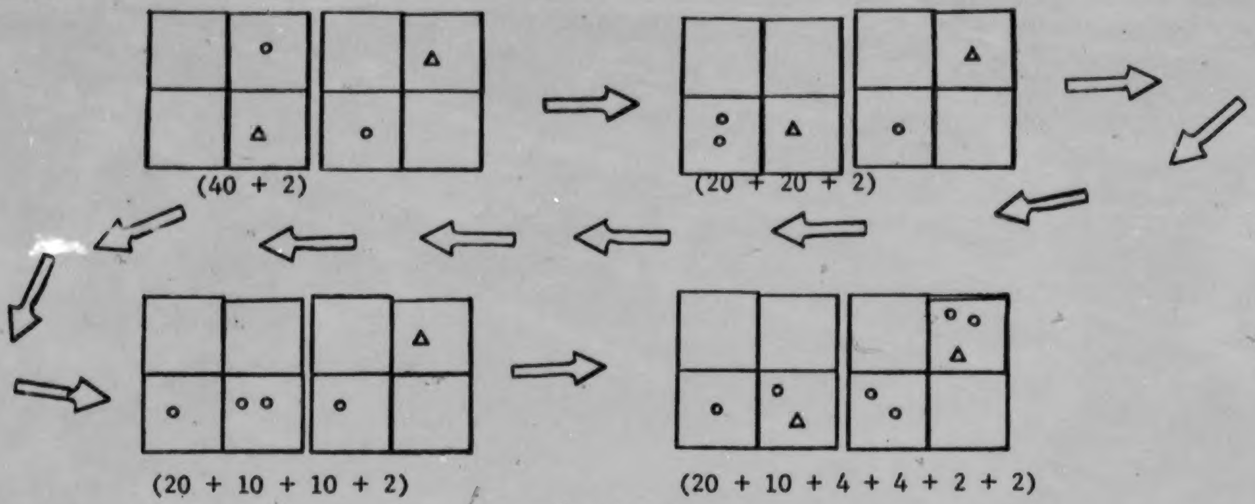
b. SUBTRACTION OPERATION:

42 - 14 = 28



(Notice we use different shaped markers ○ and △.)

STEP ONE:
"Expand" the larger number by regrouping it into successively small units until some of the ○ markers are in the same boxes as the △ markers.

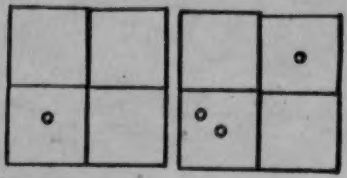


GROUND RULES FOR OPERATING THE MINI-COMPUTER: (Cont.)

5. (Cont.)

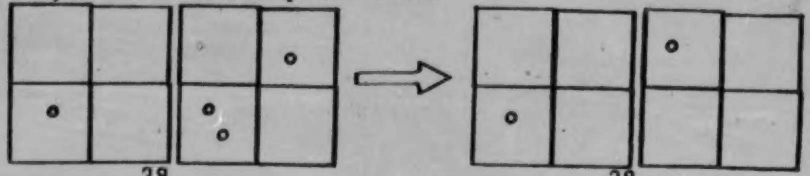
b. SUBTRACTION OPERATION: (Cont.)

STEP TWO:
The "subtraction operation" can now be carried out. This uses a "comparison method" of finding the difference. The 1-ten (o) "cancels" the 1-ten (Δ) and the 1-four (o) "cancels" the 1-four (Δ), leaving the end result or the difference:



= (20 + 4 + 2 + 2 = 28)

STEP THREE:
This step is not always necessary, but in this example is needed--"reducing the computer to its simplest form."



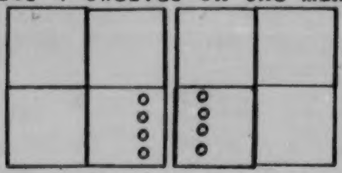
$(20 + 4 + \overset{28}{\overline{2 + 2}})$ $(20 + \overset{28}{\overline{8}})$

(We merely regroup the 1-four and 2-twos into 1-eight.)

c. MULTIPLICATION AND DIVISION OPERATIONS:
To multiply and divide on the mini-computer, we use "repeated additions" and "repeated subtractions" methods.

$$\begin{array}{r} 12 \\ \times\ 4 \\ \hline 48 \end{array}$$

STEP ONE:
Put counters for 4-twelves on the mini-computer:

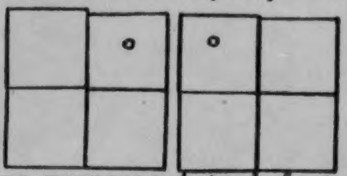


(4 tens) (4 twos)

(10 + 2)
(10 + 2)
(10 + 2)
(10 + 2)

(You will note that to "display" a 12 on the mini-computer, we first had to "expand" it to 10 + 2.)

STEP TWO:
We now seek to "simplify" the 4-tens and 4-twos by regrouping; thus. . .



= 48

(4-tens regroups into 1-forty)
(4-twos regroups into 1-eight)

GROUND RULES FOR OPERATING THE MINI-COMPUTER: (Cont.)

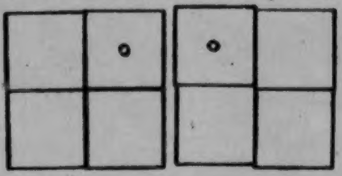
5. c. (Cont.)

Another way to look at this process is:

$$(10 + 10 + 10 + 10) + (2 + 2 + 2 + 2) = (40) + (8) = 48$$

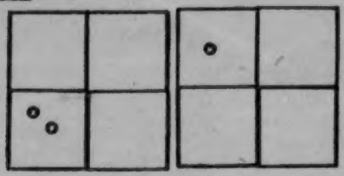
$$\begin{array}{r} ? \\ 12\overline{)48} \end{array}$$

STEP ONE:
Start by placing counters on the mini-computer to "display" 48:

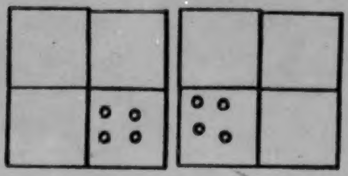


= 40 + 8 = 48

STEP TWO:
"Expand" the 40 + 8 so that you end up with counters on both the tens and twos squares:

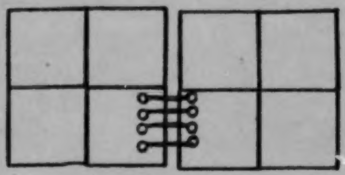


(. . .)



(NOTE: This may actually require several intermediate regroupings (. . .) but the above illustrates that the 2-twenties was regrouped into 4-tens, and the 1-eight was regrouped into 4-twos.)

STEP THREE:
This step involves "repeatedly subtracting" or at least repeatedly identifying the groups of 12 which can be regrouped from the 4-tens and 4-twos:



(10 + 2) ONE TWELVE
(10 + 2) TWO TWELVES
(10 + 2) THREE TWELVES
(10 + 2) FOUR TWELVES

One way to show "how many groups of 12 in 48" is to lay tongue depressors or colored sticks across each 10 + 2 pair. This process is an interesting way to illustrate that the numbers used in division has specific meanings:

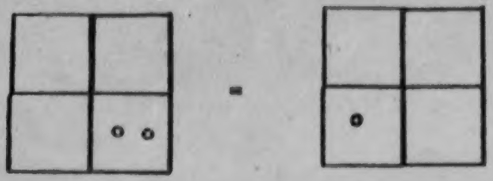
$$\begin{array}{r} 4 \\ 12\overline{)48} \end{array}$$

(number of groups of 12)
(total number of units or items)
(size of groups)

SOME GAMES TO PLAY WITH MINI-COMPUTER:

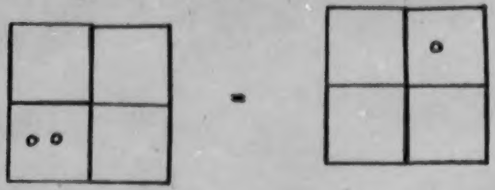
1. **TRAINS GAME:**
This game is one often used in the early stages of using the Cuisenaire Rods.

"How long is a white-white train?"



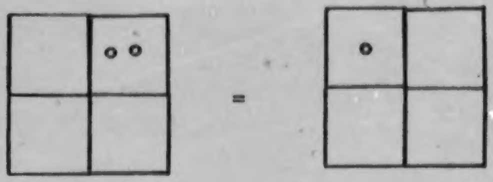
(white-white = red)

"How long is a red-red train?"



(r-r = p)

"How long is a Purple-purple train?"



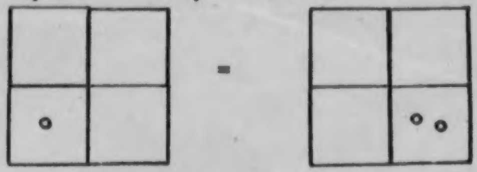
(p-p = b)

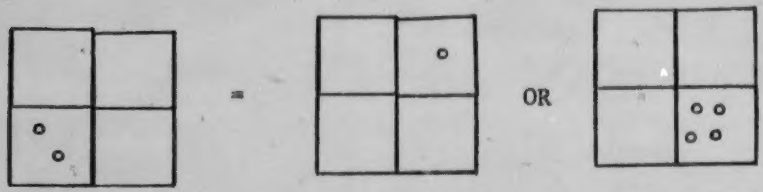
Any number of "equivalence" experiences can be gained by different train lengths and combinations.

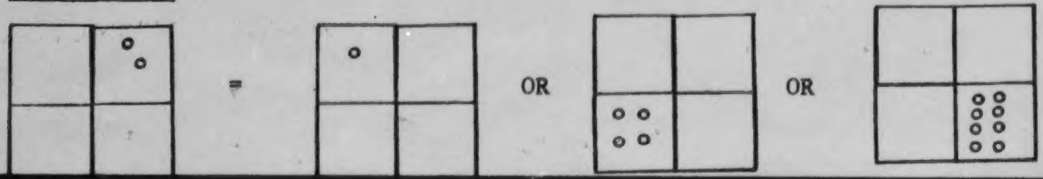
2. **OTHER "EQUIVALENCE" GAMES:**

Place 1 or more counters on a space and say . . .

"How could you regroup it?"





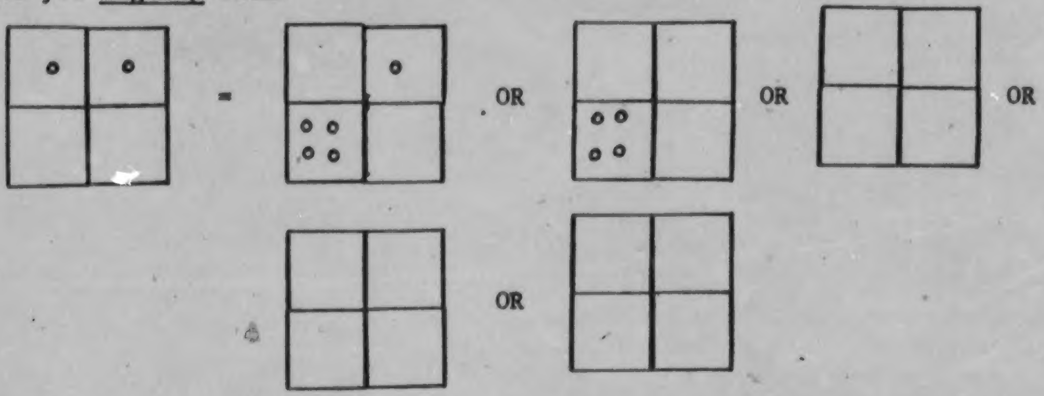


SOME GAMES TO PLAY WITH MINI-COMPUTER: (Cont.)

2. (Cont.)

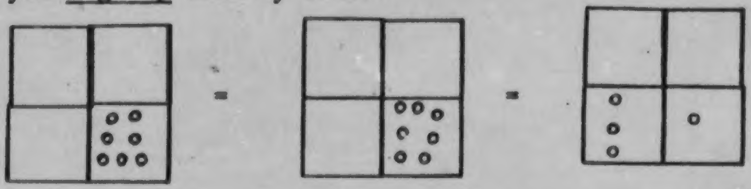
Place 2 counters on more than one space and say . . .

"How could you regroup them?"

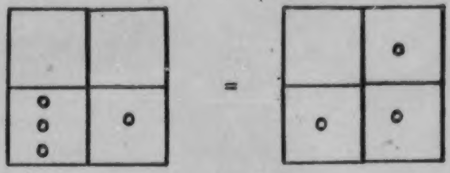


Place several counters on the space and say . . .

"How could you regroup them by twos?"

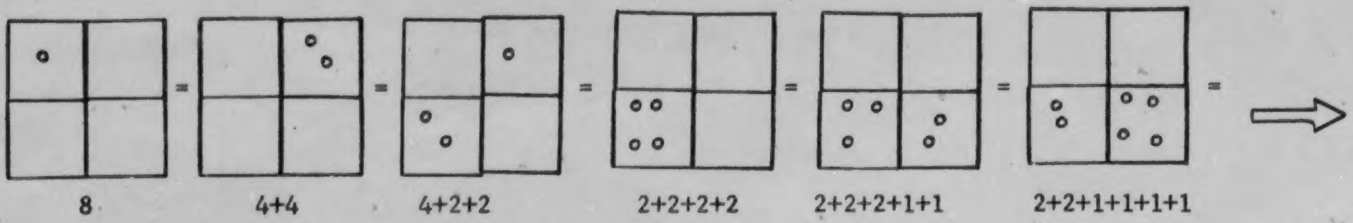


This regrouping would take place one-at-a-time--2-ones = 1 two, 2-ones = 1 two, 2-ones = 1 two. The above example may then be regrouped to "simplify" the result, thus. . .



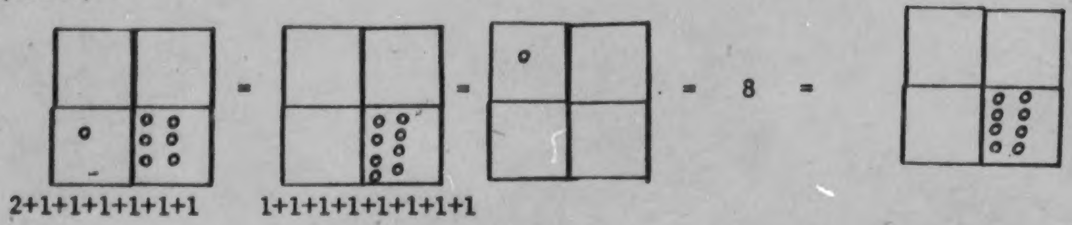
(2 + 2 + 2) + 1 = 4 + 2 + 1 = 7

To take a learner through the total process of "expanding" or "regrouping" a number you might try this. . .



SOME GAMES TO PLAY WITH MINI-COMPUTER: (Cont.)

2. (Cont.)



(This experience is cyclical, that is, it "reduces by regrouping" and then "simplifies by regrouping." This kind of experiencing is very valuable for learners in developing not only the idea of equivalency but also that numbers may be distributed in many different ways--emphasizing that a number has many names.)

3. EXPANDING THE MINI-COMPUTER--PLACE VALUE GAMES:

This exercise actually emphasizes equivalency and operations on numbers as much as it does place value notation. These games introduce a second mini-computer storage sheet which extends the "capacity" of the mini-computer. The "values" of the second storage sheet would be "multiples of 10" of the basic sheet:

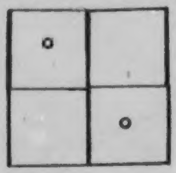
80	40	8	4
20	10	2	1

The "capacity" of the mini-computer can be expanded as much as needed and in both directions:

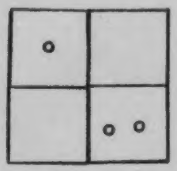
8000	4000	800	400	80	40	8	4	.8	.4
2000	1000	200	100	20	10	2	1	.2	.1

(Basic sheet)

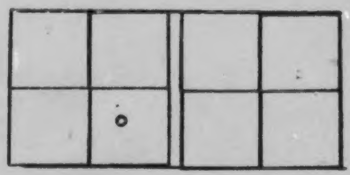
Some activities to explore this "new capacity" might be as follows:



= 9 "Add 1 to 9"

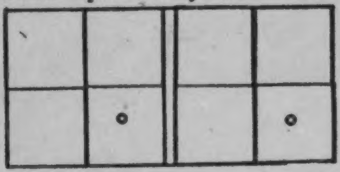


Then, add the second sheet and put this on it:

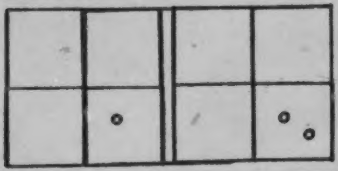


SOME GAMES TO PLAY WITH MINI-COMPUTER: (Cont.)

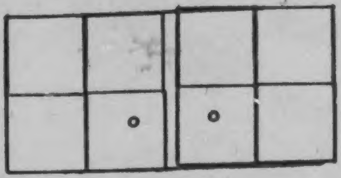
3. (Cont.)
Now, work up to say at least 19 by adding 1 to 10; 1 + 11, 1 + 12, etc. to 1 + 18. . .

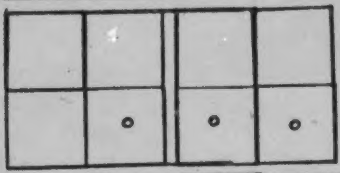


= 11



= 12 =





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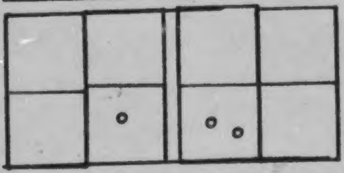
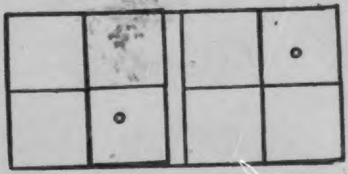
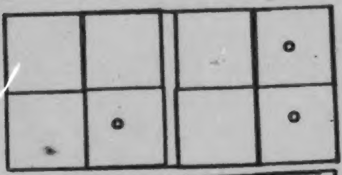
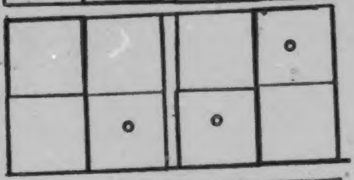


= 14 =





= 15



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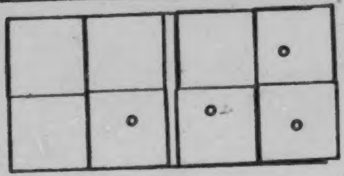
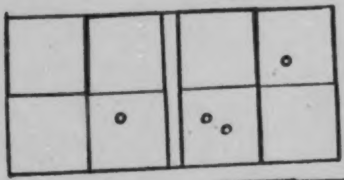
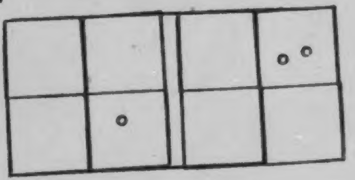


= 17



= 18 =



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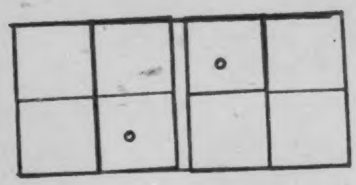
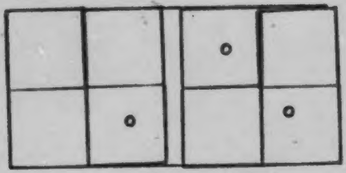


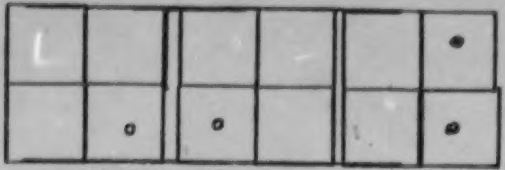


= 19 Etc.

SOME GAMES TO PLAY WITH MINI-COMPUTER: (Cont.)

4. NUMBER RECOGNITION AND ADDITION GAMES:

Since a number is "recognized by combining or putting together binarily the counters displayed on the mini-computer" this process actually represents how addition takes place. For instance the number 125 is displayed (in its simplest form) as follows:

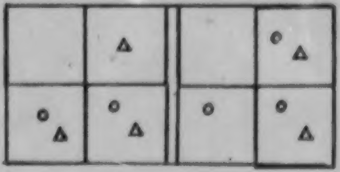


100 + 20 + 4 + 1 = 125

In order to "recognize" this as 125, the learner combines the values represented by the counters two-at-a-time thus, 100 + 20 + 4 + 1 = 125.

Another aspect of addition games occurs when you actually put 2 numbers on the mini-computer and then "combine them:"

37 + 75

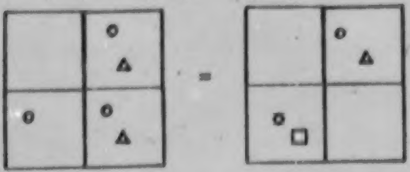


37 (o)
(20 + 10 + 4 + 2 + 1)

75 (Δ)
(40 + 20 + 10 + 4 + 1)

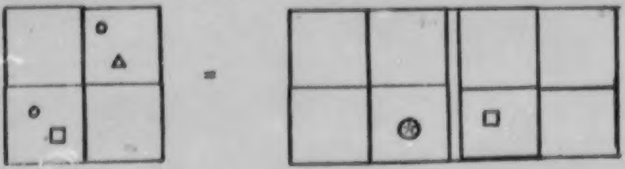
To "combine" the two numbers 37 and 75, start with the smallest values. . .

1 + 1 = 2
(o) (Δ) = (□)



(4 + 4 + 2 + 2)

2 + 4 + 4 10
(o) (o) (Δ) = ⊗

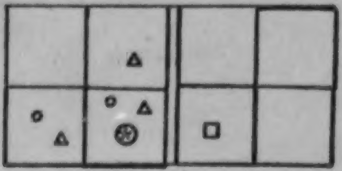


(10 + 2)

SOME GAMES TO PLAY WITH MINI-COMPUTER: (Cont.)

4. (Cont.)

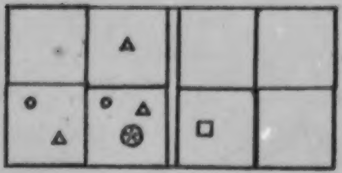
Now we have this:



(40 + 20 + 20 + 10 + 10 + 10 + 2)

20 + 20 = 40

(o) + (△) = ♯



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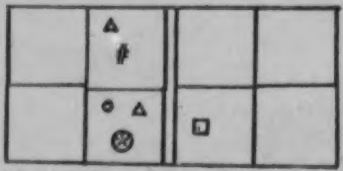
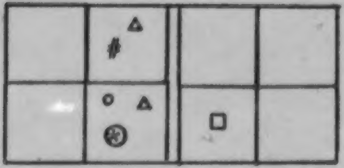


(40 + 40 + 10 + 10 + 10 + 2)

40 + 40 = 80

(△) + (♯) = ◆



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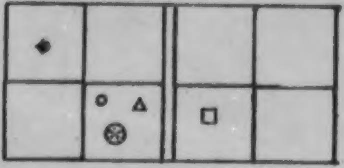
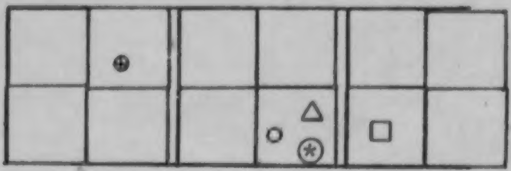


(80 + 10 + 10 + 10 + 2)

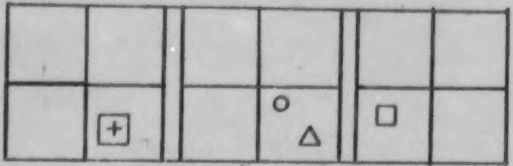
Expand the mini-computer by adding another sheet:



Then, regroup. . .

80 + 10 = 100

(◆) (⊗) ⊞

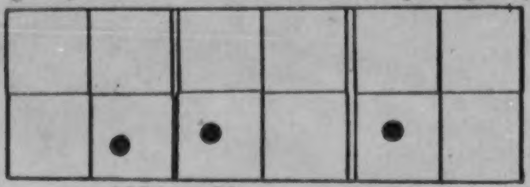


(100 + 10 + 10 + 2)

SOME GAMES TO PLAY WITH MINI-COMPUTER: (Cont.)

4. (Cont.)

"Simplify" for the final step by regrouping 10 + 10 into 1-twenty:



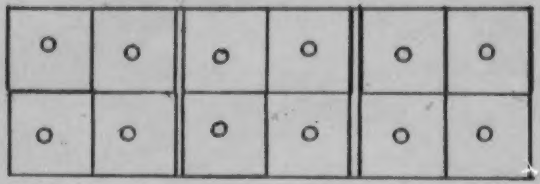
100 + 20 + 2 = 122

Therefore 37 + 75 = 122

(NOTE: The above example is only one way that this problem could be worked! There are many other ways to arrive at 122 and all not only should be allowed but accepted as the learners "work through them." However, it is the intent to encourage the learner to work toward "an efficient" way! This can often come about through increased experiences with these kinds of explorations but it is doubtful that teachers need to insist or reinforce A RIGHT WAY!)

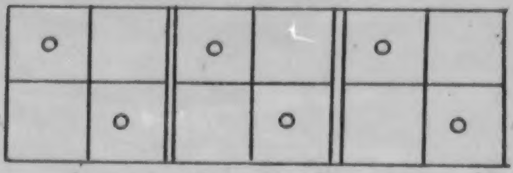
5. PUZZLE GAMES WITH THE MINI-COMPUTER:

The "patterns" made by counters placed on the mini-computer are interesting and challenging:



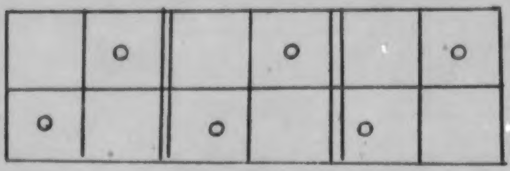
(counter in each box)

1 + 2 + 4 + 8 + 10 + 20 + 40 + 80 + 100 + 200 + 400 + 800 = ___?___



(counter in opposite corners)

1 + 8 + 10 + 80 + 100 + 800 = ___?___

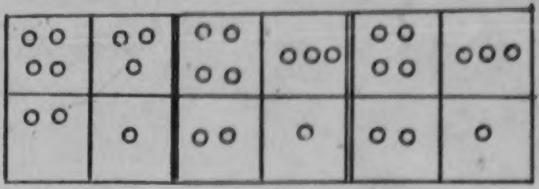


(reverse of one above)

2 + 4 + 20 + 40 + 200 + 400 = ___?___

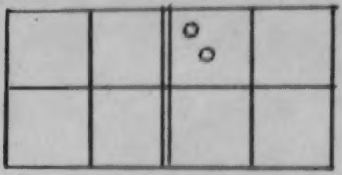
SOME GAMES TO PLAY WITH MINI-COMPUTER: (Cont.)

5. (Cont.)

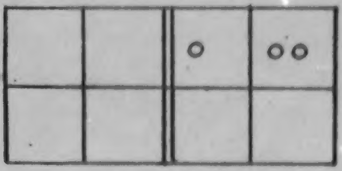


1 + (   ) + (   ) + (   ) +
10 + (   ) + (   ) + (   ) +
100 + (   ) + (   ) + (   ) = ___?___

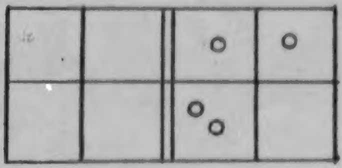
This game may cause some interesting "manipulations:"



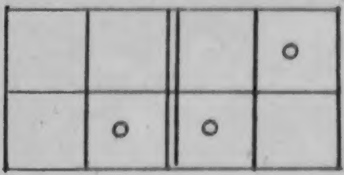
In order to "simplify" this you first need to "expand" at least one of the two eights, thus. . .



BUT, this expansion needs further expansion of 1 of the fours, thus. . .



NOW, 1-eight and 1-two can be combined into 1-ten, thus. . .



(simplest form: 8 + 8 = 16 or 10 + 4 + 2.)

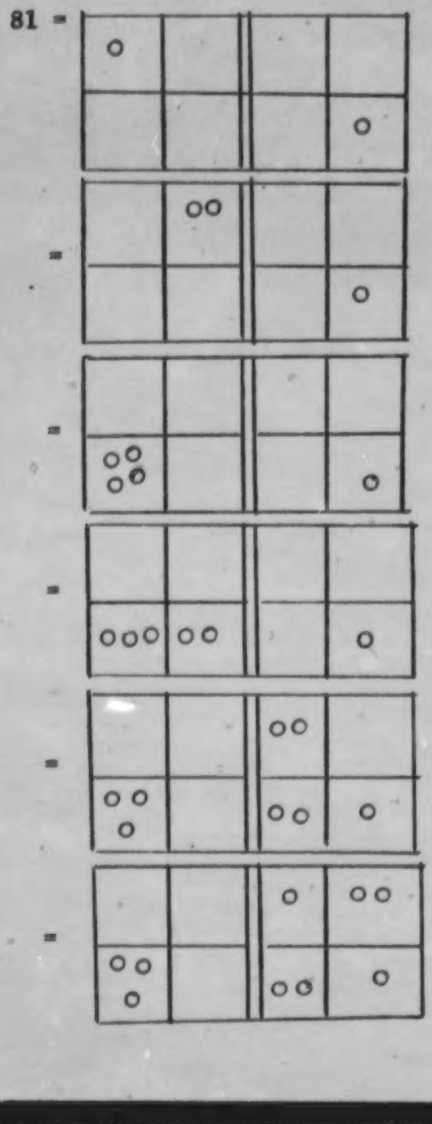
SOME GAMES TO PLAY WITH MINI-COMPUTER: (Cont.)

5. (Cont.)

A "BIG GAME" to try is this: "Put all the numbers from 1 to 19 on the same mini-computer, then "simplify" them to see if you get 190."

(Keep a record of the steps you use to "simplify" these numbers. By the way, did you get 190? There are many interesting things that can happen to a learner's thinking as he tries this one, especially if the "distributive principle" is put to work in a variety of ways.)

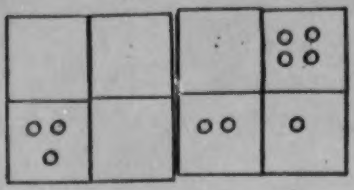
6. AN EXAMPLE OF 1/3 of 81 ON THE MINI-COMPUTER:



SOME GAMES TO PLAY WITH MINI-COMPUTER: (Cont.)

6. (Cont.)

81 =



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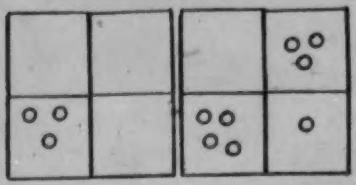

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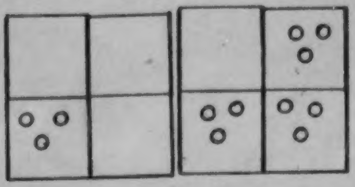
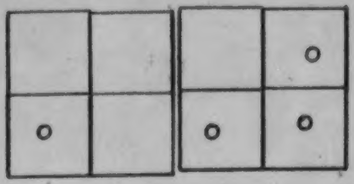


1/3 of 81 =

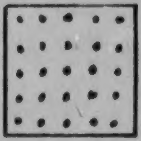


= 27

Geo-Boards

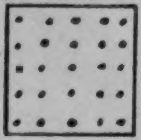
A. What is a Geo-Board?

Geo-Boards traditionally are thought of as a square board about 9" by 9" with 25 nails or pegs arranged in a 5 by 5 array:

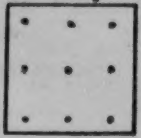


The nails allow placement of rubber bands such that outlines of various geometric shapes are formed. Unfortunately, many teachers (and, subsequently, many learners) see the Geo-Board within these very limited uses! This is unfortunate because the Geo-Board can be one of the most versatile and useful tools for a wide variety of math learning experiences.

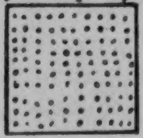
Geo-Boards can be of many varieties, shapes, and arrangements:



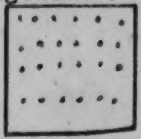
5 by 5 Array
(25 pegs)



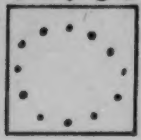
3 by 3 Array
(9 pegs)



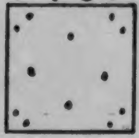
10 by 10 Array
(100 pegs)



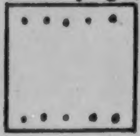
4 by 6 Array
(24 pegs)



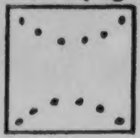
(circular)
12 pegs



(patterns)



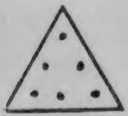
(1:1 and 1:many)



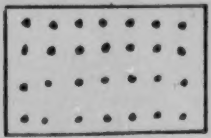
(symetrical)

Etc...

Geo-Boards can even be shaped in many ways:



(triangular)



(rectangular)

Geo-Boards can be made from many different materials:

clear plastic: (useful for overhead projection or with circular arrays of pegs on one side and rectangular arrays of pegs on the other side.)

½" to 1" thick materials:
- plywood
- forest board
- underlayment board
- veneer plywood
- solid 1" x 9" x 9" sheeting board (old school desk tops are ideal.)

pegs:
- lino-nails (round-headed at least 5/8" long)
- finish nails (headless)
- decorator nails (colored)
- dowels (holes must be drilled in boards and glue these in--about 1" long)

A. (cont.)

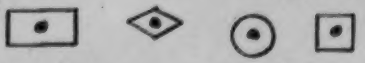
finish:

Boards may be spray painted using the canned paints, clear finish with laquer, or left unfinished.

rubber bands:

Colored rubber bands of varying lengths are best. Color is an important factor in many of the learnings with Geo-Boards. Bonds should not be so short that stretching becomes difficult for "small fingers." Neither should they be "too loose" when stretched.

variations:

Colored "markers" of varying shapes with holes punched in them can serve many useful purposes:  ETC.

B. How are Geo-Boards Used?

There are almost limitless ways that Geo-Boards can be used, but there is a suggested "preliminary" which learners should be encouraged to follow before recognizing the limitless possibilities:

FIRST STEPS:

1. Let learners examine the Geo-Boards. (For size, shape, arrangement of board and pegs, what happens when rotated, etc. AVOID definitions or giving of explanations about what the Geo-Board is or about what it can be used for. Such information will tend to set limits upon possible uses which learners will discover!)

2. Give learners rubber bands and markers and let them experiment with them on the Geo-Board in FREE PLAY. Allow them to make shapes and patterns. Such FREE PLAY requires time--don't hurry them. Learners develop intuitive understandings of many mathematical concepts and provide the basis for later uses of the Geo-Board. Periods of FREE PLAY and experimenting with the Geo-Boards should be frequent and untethered by structured activity, but learners will soon reveal readiness for ACTIVITY CARDS, involving guided-discovery and inquiry activities with the Geo-Boards.

C. Suggested Activities with Geo-Boards:

The Geo-Board is a simple tool that allows learners to:

a. visualize mathematical concepts
b. develop intuitive thinking
c. discover numerical relations and operations

Learners can develop the above for themselves--by their own hands-on involvement. The Teacher can assist such learning through carefully developed ACTIVITY CARDS. Modern educational philosophy advocates the use of many approaches and materials to help learners see mathematical relationships, to integrate rather than isolate various branches of mathematics. The Geo-Board has proved to be an ideal tool for this purpose. While it is widely accepted as a specific aid in exploring basic geometric concepts, its application is by no means limited to this area. A wide range of mathematical concepts can be investigated and clarified through Geo-Board activities.

C. (cont.)

Below are a number of Geo-Board applications that are useful with any other type of program or texts. Geo-Board applications will transform passive learning to dynamic activities leading to direct learning experiences.

D. Some Cautions:

a. Under no circumstances should learners be required to learn or recite rote definitions for concepts explored through such tools as Geo-Boards. In the process of investigating concepts via the Geo-Board, learners will come up with their own definitions--discovered on their own.

b. All learners need not be doing the same activity at the same time. They will work effectively individually or in groups with and without the Teacher present. Each learner need not do every exercise that another does.

c. All Geo-Board activities are not enrichment. Some can specifically satisfy reinforcement, exploration, or remediation.

d. The Geo-Board should not be used as a "you can play with it after work is done!" Play with a Geo-Board is more significant than many drill exercises! The use of Geo-Boards will necessitate more movement, higher sound levels, ("EUREKA, Look what I discovered!"), and more learner-to-learner interactions (learning from each other). This will require the development of higher tolerance levels by teachers for movement, sound, and sharing of discoveries.

e. Encouragement should be made to utilize such tools as Geo-Boards continuously for verifying concepts and relationships as learners are working with text assignments or when involved with problem solving. The use of Geo-Boards should not be limited to only special time use!

f. Geo-Board Activities should not be isolated from pencil-paper activities with math concepts and processes. DOT SHEETS and other devices are often helpful to use in conjunction with the Geo-Board. Some learners need various representations of processes in order to internalize them.

E. Specific Activities with Geo-Boards:

Geo-Board activities can be stimulated easily through the use of LAB ACTIVITY CARDS which present problems and activities which stimulate learners to explore and discover.
Following are ideas for LAB ACTIVITY CARDS which will contribute to exploring a wide range of math concepts with Geo-Boards. Develop some CARDS:

LAB ACTIVITY CARDS

1. SETS:
Attributes: (color, shape, size, position, number...)
One-to-one correspondence: (matching elements)
Equivalent Sets: (sets with same cardinal number.)
Non-Equivalent Sets: (more than; less than.)
Counting: (number--the concept of "how manyness" of a set. Numeral--symbols, oral, written, or pictorial representations that stand for numbers.)
Succession: (cardinal numbers; ordinal numbers...)

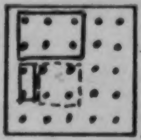
E. (cont.)

2. OPERATIONS:

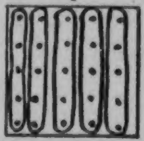
Union of Sets: (joining of 2 sets into 1. □ □ □□)

Addition: (joining together of the number attributes of 2 sets that have no elements or members in common.)

Subtraction: (some elements removed--some elements remain; comparison of more than--less than; additive concept "what added to a number equals the other number?" Example: 6 and 2--what added to 2 equals 6 or 6 = 2 + □ or on the Geo-Board:)



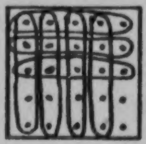
Multiplication: (repeated addition of equivalent sets:)



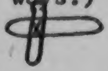
"5 fives"

(add 5 sets of five elements.)

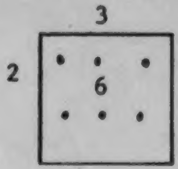
Arrays: are special illustrations of multiplications:



"3 fours"
(count the pegs "surrounded by bands stretched both ways:)

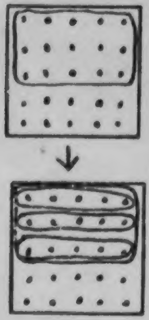


OR



(2 rows of 3 pegs.)

Division: ("what number times 3 equals 15?--repeated subtraction of equivalent sets from a larger set:)



15

5 x 3 = 15

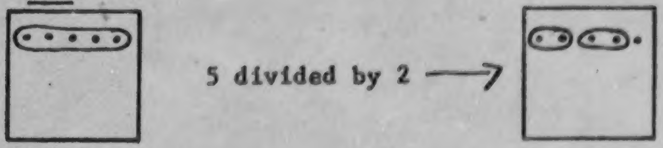
Separating a set into equivalent groups and counting the number of these groups.

E. (cont.)

2. (cont.)

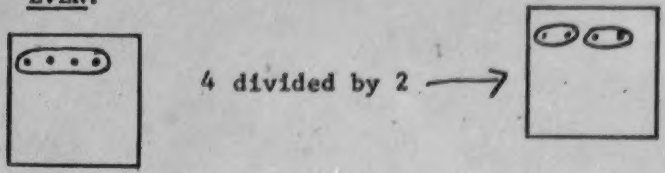
Odd and Even numbers: (Division concepts help explore ODD and EVEN:)

ODD:



5 divided by 2 → 1 left over

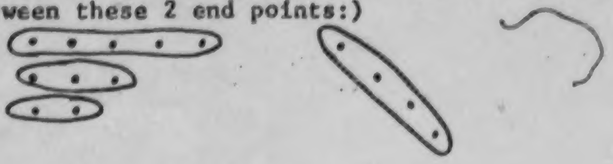
EVEN:



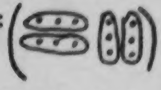
4 divided by 2 → none left over

3. LINES: (NOTE: Lines are ideas not physical objects--an infinite set of points. ⟷ Neither a rubber band stretched between 2 pegs, nor a drawing is really a line; rather each is a physical representation of a line! Points are also ideas--an exact location or position in space. A peg on a Geo-Board is a physical model of the idea of a point!)

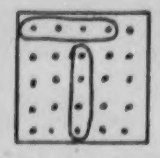
Line segments: (part of a line and consists of 2 end points and all the points on the line between these 2 end points:)



Parallel Lines:

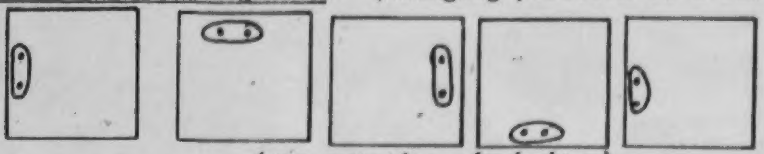


Conservation of Length:



same or different length?

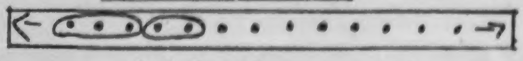
Rotation of line Segments: (changing position of a line segment:)



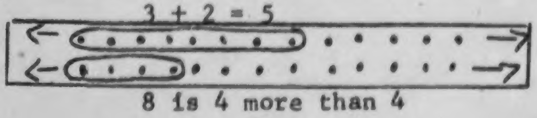
(one rotation clockwise.)

DOT PAPER helpful for use with these activities.

Number line: (Evenly spaced segments, numbered in consecutive order: This can be a special Geo-Board with 1 or more rows of pegs:)



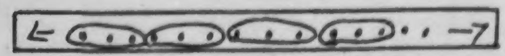
3 + 2 = 5



8 is 4 more than 4

OR

4 is 4 less than 8

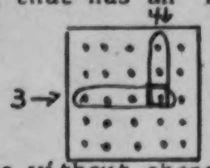


4 x 3 = 12

E. (cont.)

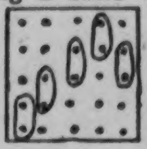
3. (cont.)

Coordinates: ("I'm thinking of a peg that has an 'address' of 3-4":)



(Learner places marker on appropriate location.)

Translation: (moving a line or shape without changing its direction:)



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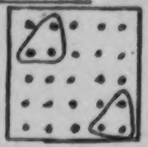
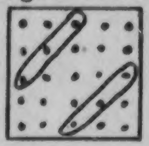


Congruent Lines: (figures that are identical in shape and size are congruent:)



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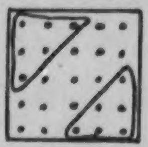
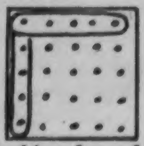
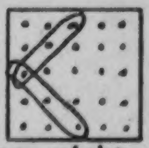


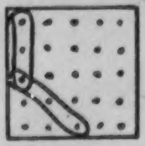
4. **ANGLES**: (formed when 2 rays meet at a point:) **KINDS OF ANGLES**:



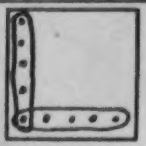
OR



OR



Perpendicular lines: (form right angle--square corner--when they meet:)



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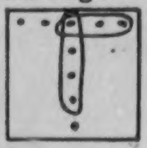
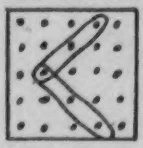
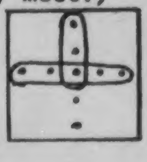


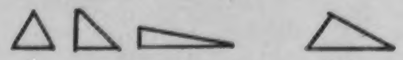
OR

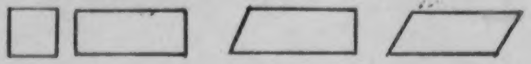


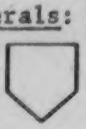
OR

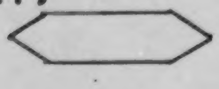


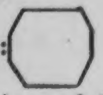
5. **POLYGONS**: (many-sided figures--3 or more:)

Triangles:  . . .

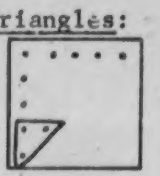
Quadralaterals:  . . .

Pentagons:  . . .

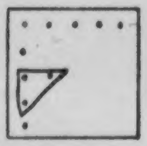
Hexagons:  . . .

Octagons:  . . .

Translation of Triangles:



TO

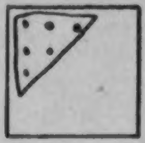


(use coordinates to translate from one position to another.)

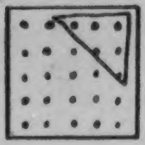
E. (cont.)

5. (cont.)

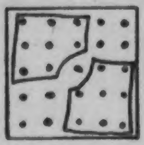
Rotation of Triangles:



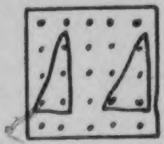
TO



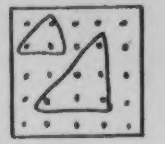
Reflection of Triangles: (mirror images of a shape:)



Congruent Triangles: (same size and shape:)

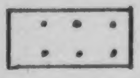


Similarity: (same shape but different size:)

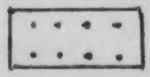


6. MEASUREMENT: (Perimeter is the length of the boundary of a shape. The surface of that shape that is inside the boundary is called the Area.)

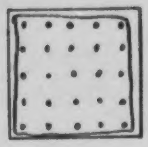
Perimeter of Rectangles:



(6 peg perimeter)

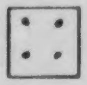


(8 peg perimeter)

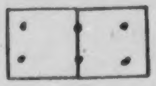


(25 peg perimeter)

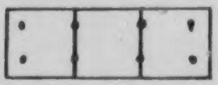
Area of rectangles:



(1 sqare unit)



(2 square units)

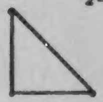


(3 square units)

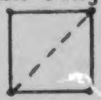
E. (cont.)

6. (cont.)

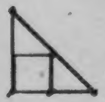
Area of a Triangle: (By using right Triangles at first, learners can explore areas of triangles easily--will discover the formula ½bh. DOT paper used in conjuction with Geo-Board helpful:)



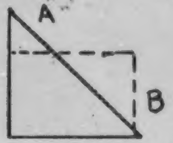
½ square unit



1 square unit or
2 ½ square units

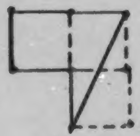


1 + ½ + ½ or
2 square units

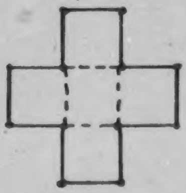


2 fractional square units equals 1 square unit. (Learners will discover congruent triangles (A & B)

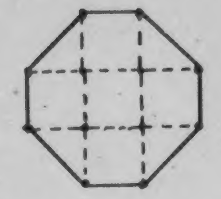
Areas of other Shapes: (Learners can be asked to find areas of shapes that contain or combine triangles and squares. Using additional colored rubber bands to mark out the squares and triangles will help them identify triangles and squares within the polygons:)



1 + 2 fractional parts = 2 square units.

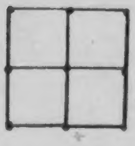


5 square units

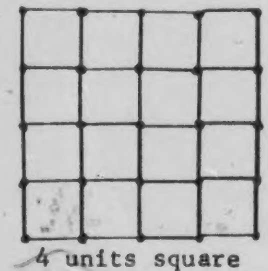


5 square units + 4 one-half square units = 7 square units.

Differences between 4 square units and 4 units square: (Some learners can be encouraged to test the difference in meanings of these kinds of statements:)



4 square units
(or 2 units square)

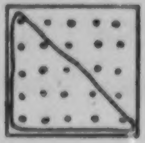


4 units square
(or 16 square units)

E. (cont.)

7. NUMBER PATTERNS: (Patterns help learners discover the structure of the number system:)

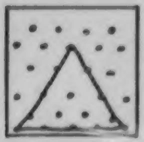
Triangular Numbers: Numbers that can be put in a triangular array, each of whose rows has one more (or less) than the preceeding row. Bowling pins and billiard balls are so arranged:



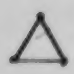
1
2 3
4 5 6
7 8 9 10
11 12 13 14 15

OR

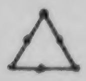
A special Geo-Board can show:



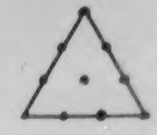
(off-set pegs)



(3 pegs)



(6 pegs)

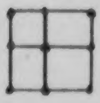


(10 pegs)

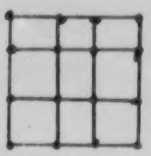
This gives a series of triangular numbers: 1, 3, 6, 10.....

Square Numbers: (can be described by a square array:)

(4 pegs)



(9 pegs)



(16 pegs)

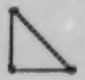
Such arrangements will, if continued, give a series of square numbers: 4, 9, 16, 25.....

E. (cont.)

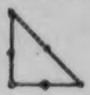
7. (cont.)Chart the data and see what it means:

Pegs per side	Total number of pegs	Differences between total numbers
1	1	3
2	4	5
3	9	7
4	16	9
5	25	(series of odd numbers)

What if you add together any 2 consecutive triangular numbers?

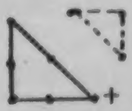


3 pegs



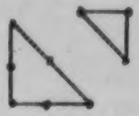
6 pegs

Now look at this another way:



3 pegs

+ 6 pegs > equals



9 pegs

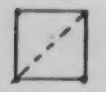
8. **FRACTIONS:** (The idea of fractions as "a part of a whole" can be discovered easily on Geo-Boards:)

(each line segment ——— is 1/3 of the whole.)

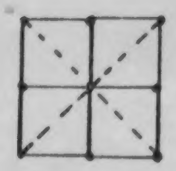
1/3

2/3

3/3

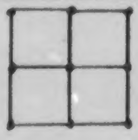


2 equivalent triangles that are 1/2 square unit each.

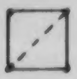


any square unit has 2 triangles, 8 triangles in all--fractions of 1/8, 2/8, 3/8, 4/8, 5/8, 6/8, 7/8, and 8/8 can be discovered. Through use of rubber bands a learner could discover 1/4, 2/4, 1/2, 3/4, and that 1/2 = 2/4, 3/4 = 6/8, 4/4 = 8/8 = 1.

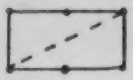
Multiplication of fractions:



1/4 + 1/4 + 1/4 + 1/4



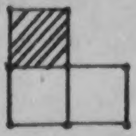
1/2 of 1/4



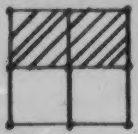
1/2 of 1/2

E. (cont.)

8. (cont.)



1/4 of 3/4



1/2 of 1 or 1/2 of 4/4

9. NOW TRY SOME CONCEPT AREAS on the Geo-Boards that are NOT identified in this Handout. Remember, FREE PLAY--emplore--experiment--provisional try-- will reveal to yourself patterns, relationships, and visualizations.

DOT SHEET

The "Colored-Arrow" game involves composing functions through use of colored arrows:

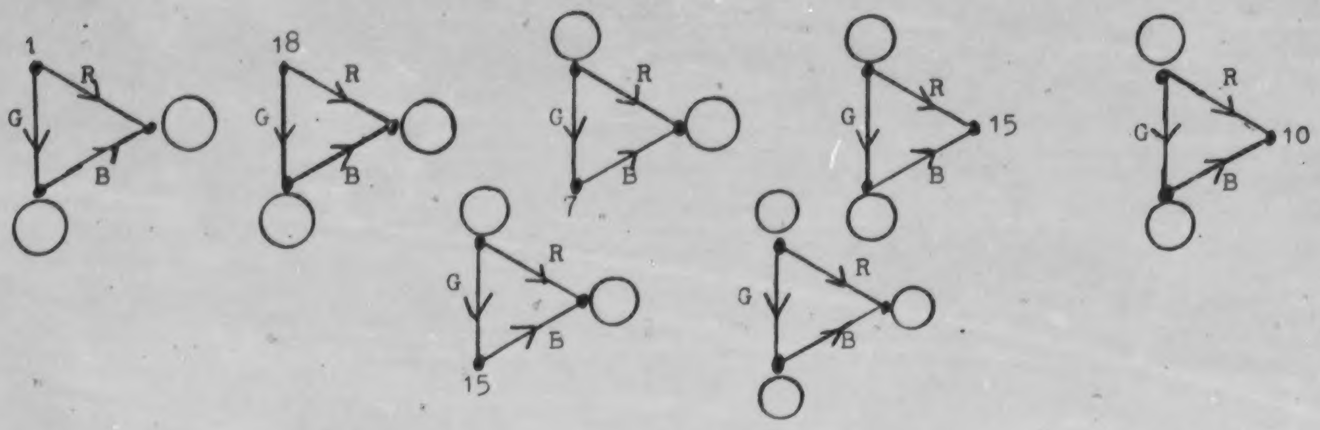
Red arrow $\longrightarrow$ = +3
Green arrow $\longrightarrow$ = +1
Blue arrow $\longrightarrow$ = +2

(NOTE: This handout does not use colored arrows due to the printing process used. Color of an arrow is indicated by : R, G, and B.

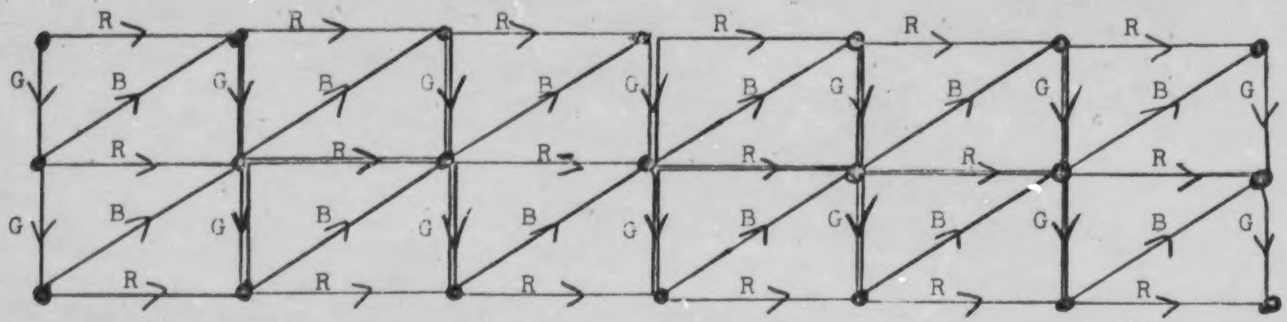
$\xrightarrow[R]{}$ for a Red Arrow
$\xrightarrow[G]{}$ for a Green Arrow
$\xrightarrow[B]{}$ for a Blue Arrow

If you make games of this type, it would be wise to do so on Overhead Transparency or color the lines.)

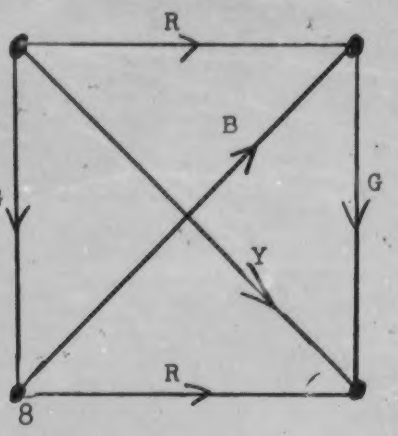
Game 1. Fill in the ears. ○



Game 2: (Place a number at each dot ●)

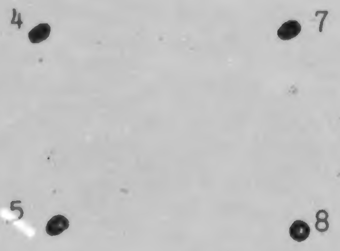


Game 3: Now add a Yellow Arrow $\xrightarrow[Y]{}$ which =+4:



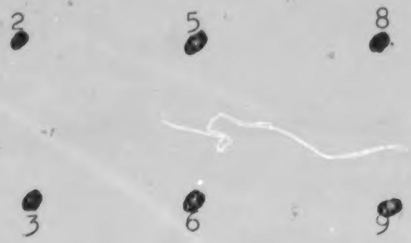
Game 4: Use: R $\longrightarrow$ G $\longrightarrow$ B $\longrightarrow$ Y $\longrightarrow$

Draw the Arrows:



Try this one: R $\longrightarrow$ G $\longrightarrow$ B $\longrightarrow$ Y $\longrightarrow$

Draw the Arrows:

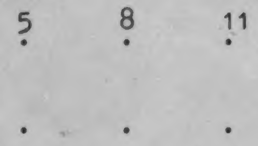


Game 5: Use $\xrightarrow[R]{}$ $\xrightarrow[G]{}$ $\xrightarrow[B]{}$ $\xrightarrow[Y]{}$; **ADD**: $\xrightarrow[BK]{}$ Black = -1

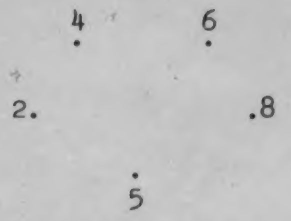
$\xrightarrow[BR]{}$ Brown = -2

$\xrightarrow[O]{}$ Orange = +5

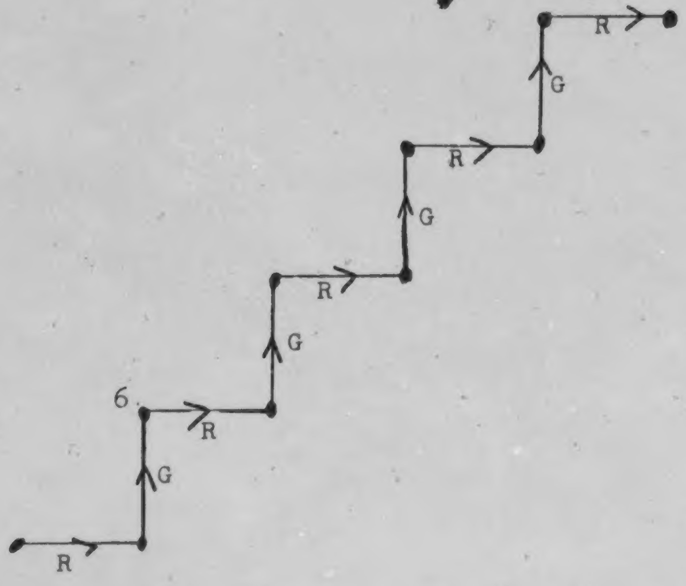
Fill in the Arrows:



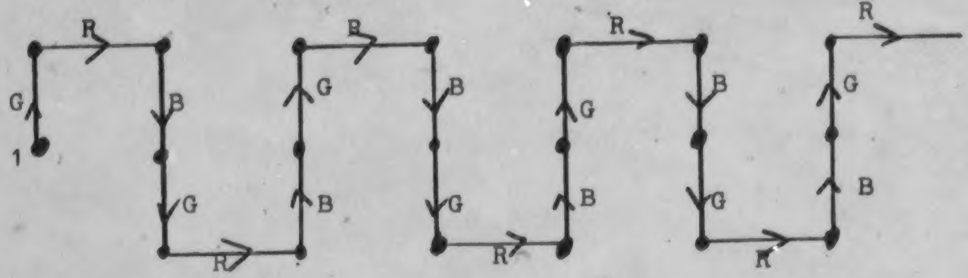
Game 6: Use $\xrightarrow[R]{}$ $\xrightarrow[G]{}$ $\xrightarrow[Y]{}$; **ADD**: Dotted Arrow ($\dashrightarrow$ = +6)



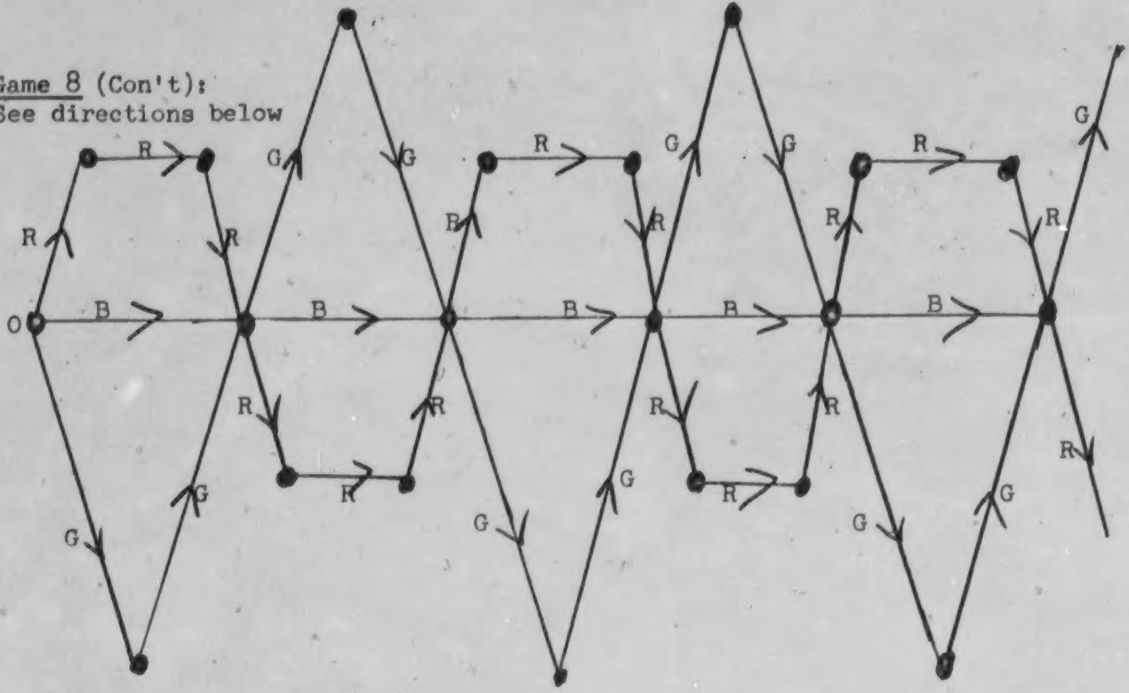
Game 7: Use $\xrightarrow[R]{}$ $\xrightarrow[G]{}$. **Draw** in the Blue $\xrightarrow[B]{}$ arrows and the numbers at each dot ●.



Game 8: Use: $\xrightarrow[R]{}$ $\xrightarrow[G]{}$ $\xrightarrow[B]{}$. Draw in the Yellow arrows $\xrightarrow[Y]{}$ and numbers at each dot (●).


1
G
R
B
G
R
G
B
B
B
G
R
R
G
B
B
G
R
R
G
B
R

Game 8 (Con't):
See directions below


0
R
R
R
B
G
G
B
R
R
R
G
B
G
R
R
B
G
G
B
R
R
R
G
R
R
B
G
G
B
R
R
G
G
R
B
R

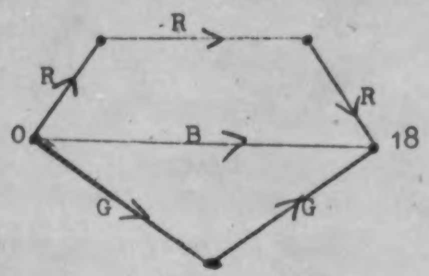
Directions for above diagram: What if you only know that $\xrightarrow[R]{}$ = +2:

Can you draw in the numbers at each dot (●)?

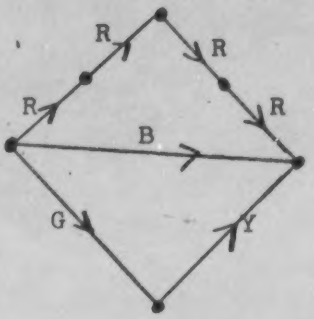
What do the Blue and Green arrows have to say?

(NOTE: Study the length and way in which the Blue and Green arrows are drawn!
Hint: What values are the Blue arrows and Green arrows in relation to the Red arrows?

Game 9: Try to determine the value of the $\xrightarrow[R]{}$ $\xrightarrow[G]{}$ $\xrightarrow[B]{}$ in this one:

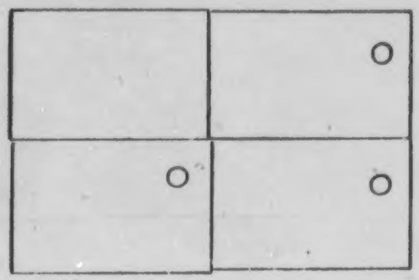


Game 10: Try solving this game, but be careful of the Green $\xrightarrow[G]{}$ and Yellow $\xrightarrow[Y]{}$.
(Hint: you can decide on values you need for each color, but two colors may not have the same value.)

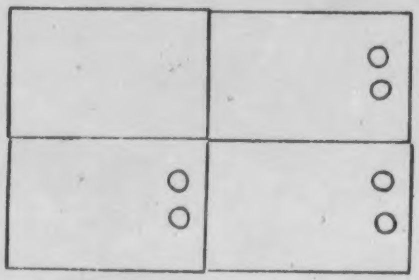


Game 11:

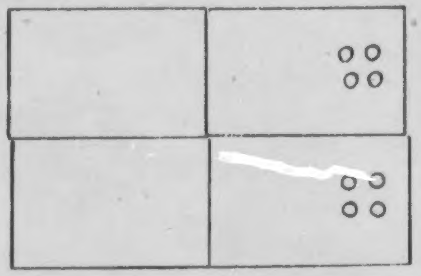
1. Start with 7 on the mini-computer:



2. DOUBLE it:

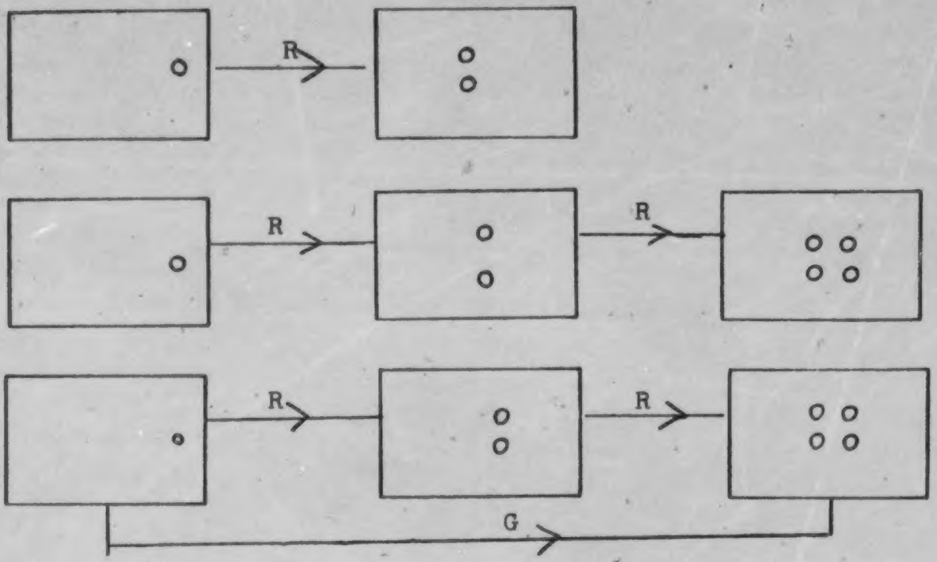


3. DOUBLE it again:

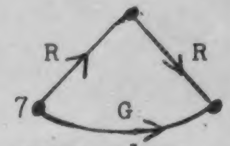


Game 11: (Continued) Now look at it another way:

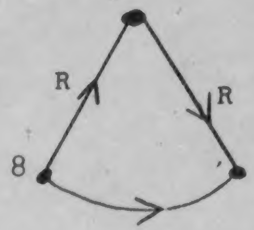
Let Red $\underset{R}{\underline{\hspace{3em}}}$ = 2 x ; What does the Green arrow mean?



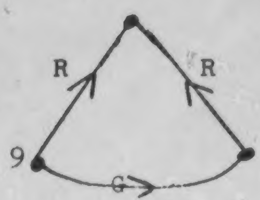
Now, look at it still another way: $\xrightarrow[R]{}$ = 2 x and $\xrightarrow[G]{}$ = 4 x (or did you answer that above?) Write the numbers beside each dot (●).



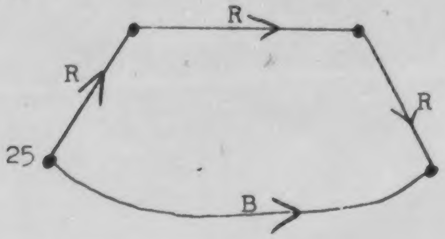
Try this one: (Same arrow value: $\xrightarrow[R]{}$ = 2 x; $\xrightarrow[G]{}$ = 4 x)



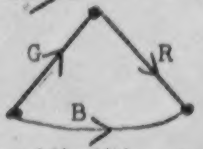
And this:



Can you "expand" to this one? (Note: What is the value of Blue ______?)

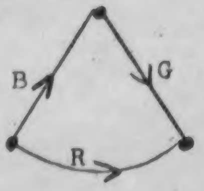


Game 12: Use: G → = +5 ; R → = +7 ; Find: B → = ?

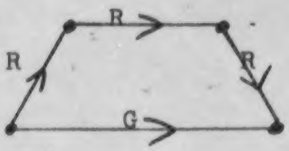


(Note: Did you have a problem with this one? What was it?)

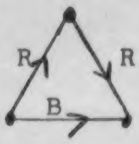
Try this one: B → = +10; G → = -3; Find: R → = ?



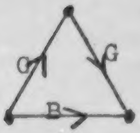
Game 13: R → = +4; Find: Green G → = ?



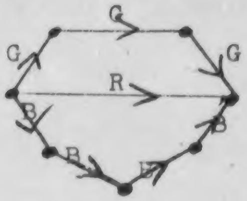
Now try this one: B → = +14; Find: Red R → = ?



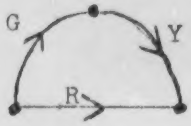
And this one: B → = +1; Find: G → = ?



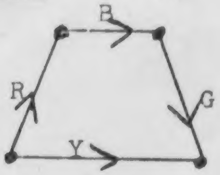
Game 14: G → = +4; Find: R → = ? and B → = ?



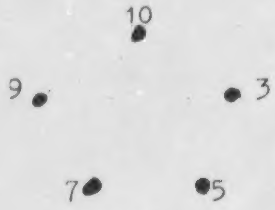
Game 15: G → = -4; Y → = +12; Find: R → = ?



Game 16: R → = +4; B → = +5; G → = +6; Find: Y → = ?



Game 17: Greater than > Less than < ; Draw G → arrows to show > than:



Education is Child's Play *

by

Dr. Clark C. Abt

Introduction

Animals and humans love to explore, to try out, to be the cause of effects, even when all primary instincts are well satisfied. The universal striving for competence is probably not powered by instinctive energies any more than muscles are "powered" by the will. It is more a matter of evolutionary adaption. Biological survival has required adaptation, which in turn required problem solving. Problem solving required a propensity to explore. Mental as well as physical muscles are most effective when continually exercised, regardless of application. Thus the mentally exercising or exploring animals tended to survive, and we can observe this pervasive propensity in animals and humans today.

Anthropology, learning theory, military and systems technology, and social needs all converge on play, simulation, and gaming as an ascendant educational technique. Anthropological research indicates the prevalence of minetic play undirected by adults as a primary educational process. Learning theory has found learning dependent on active participation, reinforcement, feedback, variety, comprehension, and perceived relevance, all of which occur in games. Military and systems technology have made both necessary and possible the modeling and imulation of very large scale complex processes. The social needs for higher quality mass education for diverse student cultural and ability backgrounds have made role plays, gaming, modeling, and simulation the most cost-effective educational innovation available today.

This educational innovation is a re-discovery, rather than a discovery. We are finally beginning to do consciously and systematically what children and adults have been doing very effectively, but intuitively and informally, for ages. The systematic application of games and simulations to education promises not only to improve the educational scope and effectiveness of these new techniques themselves, but also suggest pervasive changes in our overall concepts of education.

Under the external pressures of international politico-military competition, and the internal strains of social change through racial integration and anti-poverty programs, our education system is being asked simultaneously to improve its quality, coverage, quantity, and minimum standards. Although much money is available in absolute terms, the amount seems small in the face of these expanding and urgent requirements. Any innovations that can greatly increase education system coverage and quality quickly and cheaply are of the highest priority. Educational games are such an innovation.

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There are some serious limitations of conventional educational methods that deny the maximum possible educational benefits to both our disadvantaged and our priviledged population of students. These limitations of the conventional methods are in the areas of motivation, intellectual initiative, structural understanding of fundamental relationships and organizing concepts, creativity and intuition, and negotiating skills. It is the argument of this essay that the inadequacies of conventional teaching techniques in these areas can be remedied at very modest cost by the use of educational games and simulations.

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* ABT. Associates Inc, 55 Wheeler Street, Cambridge, Massachusetts.

Limitations of Conventional Educational Methods:

The objective of education is to bring students to a state of mastery over facts, problem-solving methods, general ideas, and the learning process itself by which this mastery is achieved and expanded. From a systems-analytic point of view, the student needs to store data, problem-sensing and problem-solving techniques, general theories, and heuristics. For the student to receive, retain, and respond to problems with this information, he must be interested in the information, he must organize it in his memory, and he must be conditioned to use it. To be interested, the student must be motivated to learn. To organize information for effective retrieval, he must have intellectual initiative and structural understanding of fundamental relationships and organizing concepts. To use information in problem solving, he must have the intuition to see the analogical relationships between what he knows and what he does not yet know ("transfer"), and the negotiating skills to apply his knowledge in a social context.

Consider current conventional education methods from the aspect of the above criteria. Motivation is the first problem. The high incidence of school dropouts in disadvantaged populations and underachievement in privileged student populations indicates many cases of inadequate student motivation. Motivation is a function of both immmediated pleasure and perceived utility in gaining future pleasure and reducing future pain.

The immediate pleasures of learning are difficult to define, but they seem to have much to do with novelty, epiphany, and mastery. The experience of a dramatic revelation of a new phenomenon, or the surprising significance of a familiar relationship, is usually pleasurable. Children and adults both smile with pleasure at surprising concepts, provided they are relevant to previous interests. The pleasure of mastery, of understanding or knowing what or how, is probably related to the universal pleasure men and animals feel in exercising and expressing their capabilities. It is the exhilarating awareness of the effective functioning of one's faculties. Animals and humans love to explore, to try things out, to be the cause of effects. Games provide this kind of participant pleasure, but listening to a lecture often does not.

The perceived utility of gaining future pleasure and reducing future pain is the more thoughtful component of motivation for learning. It is dependent on the student's perception of the relevance of what he is learning to his own expected future life. "Relevance" tends to be perceived most in things related to immediate objectives. Thus relevance is very dependent on the social context and age of the student. Academic knowledge is often perceived as irrelevant by poor children, because they have no expectation of using it in going to college or with their parents. To a child in elementary school, a series of dry facts such as historic dates is irrelevant to his perceived objectives. The irrelevant offers no utility for the achievement of future pleasure or reduction of pain.

The contents of games are inferently relevant to any participating players, because to achieve their immediate objectives they must know these contents. If students enjoy a game at all, they must be interested in its content. If that content consists of facts, processes, and concepts to be learned, then the student players will try to learn these simply to perform well in the game.

Recall and application of learned information requires the learned information to be organized and related to other stored information. In this way, the searching focus of attention can, by connecting with the related information, trace a logical path to that which was to be recalled. It seems probable that the more information contexts the recently learned information has been consciously connected with, the greater is the probability of a given scanning of the memory recalling that recently learned informaion. The more connections that are made between recently learned and stored material, the better the recall.

This conscious connecting of recently learned information with much stored material requires the organization of the information into types and categories, which can then be individually connected to other related, already stored material. This organization of the learned information--or, more precisely, this dis- and re-aggregation of it--requires a structural understanding of the fundamental relationships involved. This structural understanding requires intellectual initiative to develop, since it is usually achieved by the conscious and frequent manipulation of the fundamental relationships.

The rote memorization of many facts and theories, without comprehension of their interrelations and relevance to other already stored information, results in short memory and difficult and unreliable retrieval of the learned information. Too much of the data presented to students for their memorization is not related to other, previously learned data. When no connection to other stored data is stated or perceived, the memorized data is often soon forgotten.

In educational games, the initiative to organize information is motivated by the interest in the game and the competitive pressures to exploit all available information for achieveing immediate game objectives. The connections to other already stored information is seen first, because it is the reason for the new information being sought, so that memorization here is the opposite of rote, being immediately needed, functionally relevant, and exercised.

The identification of information significant for the solution of given problems is often a matter of intuition. So is the sudden insight into how a set of apparently disparate facts are related in a pattern. Systematic attempts to recognize patterns, relationships, and relevant information can be and should be made, but they are rarely as efficient as the unconscious scanning we call intuition or insight. Much of our conventional formal education is highly structures, standardized, and rigid--no doubt for ease of administration by mediocre teachers pressed for time. While much structured teaching must be tolerated for practical reasons, it often becomes counterproductive when it denies opportunities for the exercise of intuition and insight. The very functional values expressed in may aspects of formal education stress solving problems by routinized, "cookbook" methods, and penalize originality of method (found intuitively) as disruptive nonconformity.

Educational games reward rather than penalize intuitive insights into problems, because results rather than methods are stressed. An intuitive solution to a game problem is superior to a doctrinaire one, when it is faster or more effective. Whenever insight outruns routine analysis, it is automatically preferred for its superior problem solving efficiency, and is thus encouraged. On the other hand, intuitive solutions found for the sake of intuition alone must stand the operational test of being more effective than analytic, procedural methods. If they are not, they will automatically be rejected for the more conscious, systematic method which is most effective in some cases. The games, like reality, provide operational tests of the most effective methods of solution, without prejudice to either conscious or intuitive means. In this sense they encourage effective insights precisely to the extent that they are effective.

The solution to many of the problems of life is not purely individual, internal, and abstract, but often also requires communication to and persuasion of others that one's personal solution is also desirable or acceptable to them. This requires skills or perception, self-expression and negotiation in interpersonal relations that are rarely taught at all in formal education. Typically, the classroom in its formal state contrains a bi-polar relationship between superior monolithic teacher and inferior, aggregated but not allied or coordinated student body. This is a poor paradigm indeed for the interpersonal complexities of negotiated cooperation and conflict among peers, superiors, and subordinates extant in most real-life problem solving contexts.

Only occasionally with conscious intent, the burden of education in social skills is placed on the family and recreational activities such as games and social events. The family--particularly the small or disrupted family--is very often inadequate for teaching acceptable social skills. Sports, games, and parties are helpful, but not available or attractive to all students in sufficient degree. Educational games can provide much of the missing training in social skills. They can change the classroom, at least some of the time, from a lecture auditorium to a marketplace of ideas and a laboratory for cooperative experiments in knowledge.

The Use of Games and Simulations as a More Serious Form of Education:

Games are considered frivolous by those holders of the Puritan or Germanic ethic that pleasurable activities are not serious, that true seriousness requires sobriety and even a little pain. We would like to use the word "serious" in the objective sense of meaningfulness and significance, rather than the subjective sense of personal attitudes. The Greek comedies of Aristophanes are subjectively humorous but most serious in significance. It is that kind of seriousness that can be provided by educational games, however much fun they may be to play.

We are accustomed to regarding certain types of play or simulation as serious subjectively too. Examples are "serious" plays in the theater, and serious religious ceremonies. Dramatic arts are often objectively serious in meaningfulness and subjectively humorous in expressed attitudes, but they can be serious in both respects. Games and simulations can also be serious both objectively and subjectively.

What makes at least some educational games and simulations more serious than conventional forms of education, however, is their greater density of meaning and significance. If seriousness in education can be construed as density of information and structural meaning, then educational games are generally more serious than other educational modes. The proven intense involvement of students in educational games is also a very serious matter, because it can change a student's entire attitude toward school itself. The student's image of school can be changed from one of a tedious workhouse not for him except by coercion, to one of a fascinating laboratory offering opportunities for the joyful expression of his own capacities.

By an educational "game", I mean a situation of multi-player mixed conflict and cooperation simulating a process or series of events which is to be understood by the student. A game is a human player simulation of a dynamic model of some abstract, symbolic, or real-world process. The uncertain outcome depends primarily on player factual knowledge, structural comprehension, and communication and negotiating skill. In our games, the only physical materials used are game boards (often with maps and sometimes with analytic charts), situation cards, symbolic counters of various kinds, and printed instructions and background scenarios. We have observed that self-directed learning in games occurs in three usually successive phases as a result of the active participation and intense involvement of the student:

I -- Learning the facts which the game context and dynamics presents

II -- Learning the processes of variable changes over time simulated by the game

III -- Learning the relative costs and benefits, risks and potential rewards of alternative strategies of decision-making

We have observed that because these three levels of game learning can occur simultaneously in multi-player teams, individual games can accommodate a very broad range of student age and achievement levels (for example, reading levels from grade 4 to grade 12). The slower students also learn from the faster ones. We have also found that culturally deprived students respond relatively much better to game teaching than to less dynamic, more expository methods. For this and other reasons, we believe that games can test the comprehension of a complex problem and its solution better than purely verbal tests, as well as offering highly motivated self-directed learning.

The games method of education includes the following characteristics:

1. A combination of the dramatic arts and the systems sciences--drama for involvement and expression, the systems approach for analysis.

2. Emphasis on developing analytic approaches and organizing concepts transferable to other problems identified by students (e.g., coordinated sequential operations, tradeoffs of competing forces, hierarchical structuring).

3. Encouragement of intuitive thinking as well as analysis by use of analogy, testing of limiting conditions, and visual expression of solutions.

4. Attempts to make learning entertaining and relevant to student's life experiences.

5. No "talking down" to students--use of realistic adult materials.

6. Learning by exploratory problem-solving simulations (games) involving role play, with self-directed student participation.

7. Communications and negotiations skills developed by team activities.

8. A cross-disciplinary, concrete experiential view of problems.

9. Abstraction capabilities built on multiple sensory experiences.

Educational games use rather than suppress the student's way of viewing things. They present concrete problems in a simplified but dramatic form that mediates between the dryness of abstraction and the confusion of multi-variable reality. For elementary school children, educational games translate the child's still primarily concrete, intuitive thinking into a sequence of dramatized possibilities that expands their awareness of hypothetical alternatives and fundamental relations! The child deeply involved in concrete game activity becomes gradually aware of the formal relationships he is manipulating, and obtains immediate pleasurable rewards for manipulating them effectively--that is, in a direction tending toward his game objectives. This increasing awareness of formal relations is the beginning of adult logical thought.

Elementary school children tend to focus on only one aspect of a phenomenon at a time, greatly limiting their ability to comprehend phenomena with even a few interactions among elements. Games present simultaneously progressing multiple interactions that can first be examined one at a time, and then gradually longer with increasing comprehensibility.

Probability, plausibility, certainty, and impossibility are concepts difficult for elementary school children to grasp from conventional media. Certainly direct verbal exposition is ineffective, because the children have not usually experienced the phenomena described and thus cannot make the connection between abstract concept and any direct experience of their own! Educational (and other) games often use probabilistic mechanisms to simulate subordinate causal sequences too complex to replicate directly. The natural interest of the children playing the games in these decisive probabilistic mechanisms leads to their understanding simple probability from a series of direct experiences. This awareness of probability, together with game pressures to make decisions based on inadequate data, leads children to develop logical strategies taking account of both probabilities and costs. This is the essence of modern statistical decision theory and cost-effectiveness analysis, and elementary school children can learn it playing games. Educational games offer opportunities for exploration that are sheltered from any irreversible consequences, thus encouraging risky attempts while yet providing realistic evaluations of their effectiveness.

The attention span of elementary school children can be stretched painlessly with the use of educational games. Attention is a function of both initial motivation and what might be called sustaining motivation. Games generate potent initial motivation due to the expectation of pleasure children and others associate with them, and because of their inherent dramatic interest deriving from action, conflict and uncertainty of outcome. The sustaining motivation is provided by the response of the game itself, and the other players, to the actions of the student player. He feels himself a cause of events, and that is apparently a universally pleasurable feeling.

The growing sense of structure among the game variables gained by the student player, with its correspondingly growing sense of structure of the subject simulated by the game, greatly expands the attention span of the student player. The more densely packed a game is with such structure (up to a surprisingly high degree of apparent confusion), the longer the learning episode that can be tolerated by the student without fatigue or loss of interest. The longer and more concentrated the learning episode, the greater the student's understanding and confidence in the intellectual satisfactions of subsequent episodes.

For secondary school students, one of the main problems is their sense of the relevance of what they are learning to their future expectations. This is not a matter of supplying a spurious immediate relevance by external rewards for academic achievement such as status and admission to coveted colleges or careers. Motivation must be sustained beyond these transient--and sometimes ephemeral--rewards. Students must believe, and believe correctly, that what they learn will be useful to them as adults. Utility here should be defined broadly, to include not only practical career guidance and training, but also the appreciation of general intellectual and social values. Students should have reason to believe that what they learn will help them to understand, predict, and control to a socially acceptable degree their own future environment, as well as their own actions in it. While there is always some connection between this intellectual utility and almost anything that can be learned, often the relative utility of school experiences compared to extracurricular experiences perceived by the student is too low to motivate effective learning.

Games and simulations that model reality can present the great issues and problems of contemporary society on a level of specific human action that directly relates the student's decisions to the larger world. The relevance of educational games perceived by students is both substantive and methodological. Games dealing with economic, political, social, and scientific problems on the adult scale are of obvious substantive relevance to the future operations in the adult world. But educational games and simulations also offer methodological aid in general problem solving. Both systematic rational cost-benefit calculations in the face of uncertainty, and intuitive heuristics are exercised in educational games, and their transfer to the solution of other systems problems in an obvious application.

A pervasive problem at all levels is the relative inflexibility of conventional educational media in simultaneously dealing with a broad range of student capabilities. The problem is not solved by segregation according to academic performance--what has been called a "meritocracy." This only narrows the social experience of the segregated groups that must in any case deal with one another after graduation, and depresses the quality of teaching available to the slow learners. The problem can be solved by educational games, in which slow and rapid learners can play together on balanced teams and learn analysis and patience from each other respectively. We have successfully played complex socio-economic games with groups within which the achievement levels ranged from first to eighth grade. Students of different capacities learn different things from the same game. Slow learners may still be learning the facts and the processes embedded in the game, while rapid learners have progressed from analysis of processes to comparison of alternative strategies.. The important point is that both slow and rapid learners can share social interactions in the game while learning from it at quite different levels.

Finally, educational games are a way of teaching analytic problem solving that incorporates the human aspects. In conventional school situations, the solution to problems is taught on an abstract, impersonal basis. In educational games, to achieve his aims a player needs not only to calculate his best move, but also to persuade his teammates of the effectiveness of the move. The student player learns loyalty and the decent limits of rivalry. The compressed competitive experience give students a realistic forecast of nature competition and negotiation, where technical success can be spoiled by social neglect or greed. Over aggressive, uncontrolled, or apathetic behavior is punished in a non-fatal way in games, disciplining and institutionalizing behavior through peer interactions. The intellectual and the social skills needed to solve adult socio-economic problems are developed in concert in educational games, as they must be applied in concert in adult life.

A Typology of Educational Games:

Teaching aids have been catagorized by Bruner (1960) into (1) devices for vicarious experience such as films, TV, recordings, and books; (2) devices for structural comprehension or model devices, such as laboratory experiments, models, and diagrams; (3) dramatizing devices such as plays, role plays, and games; and (4) automatizing devices such as teaching machines and programmed instruction. Educational games are a combination of all of these types of teaching aids in varying degrees, as well as incorporating some conventional expository teaching among players.

We would prefer not to classify educational games according to their provision of vicarious experiences, structural comprehension, drama, and automaticity, because many educational games have all these characteristics. Rather, the game characteristics themselves suggest a four-fold typology that seems more discriminating with respect to the games themselves.

Games are contests of skill and chance among men or between men and nature. They also incorporate varying degrees of reality and fantasy, paralled skill and chance. These two dimensions of games suggest the corresponding tensions in between control and submission to nature, calculation and spontaneity, action and passively being acted on, ego regulation and libidinal imagery, realization and potentiality. Games of skill such as chess or tennis are at the control end of these paired characteristics. Games of chance such as dice and roulette are at the submission-to-nature end. (See Table I of characteristics of four types of games.)

Another dimension is that of abstraction to richness, or symbol to substance. Thus games of both skill and chance such as chess and dice are abstract and symbolic, while games of both reality and fantasy such as war games and dances are rich in their substantive contents.

A descriptive distinction can also be made between showdown games and strategic games. Showdown games may be games of chance like dice, skill like discus throwing, or (more rarely) reality like fortune hunting, but they all are contests in which adversaries operate autonomously. Strategic games also include games of chance like poker, skill like chess, or reality like monopoly, but in these games the adversaries affect each other's fortunes. Educational games are usually strategic games, precisely because the interactions among adversaries offer the most opportunities for simulating the interaction of the elements of a complex problem.

Typologies are useful in discriminating among different types, so that their inter-relations and applications can be identified more precisely. The table of characteristics of four types suggest the interaction between games of skill, chance, reality, and fantasy. Games of chance tend to become games of skill with repeated play, as players learn the relative probabilities of game events and maximize their gains by the correct betting policies. Luck is reduced to a minimum and, for many players, becomes secondary to skill. The good poker player may not be able to predict the outcome of every hand, but may establish a predictable pattern of winning.

Games of reality, or simulations of some real-world process (e.g., "Monopoly" as a imulation of business competition) also tend to become games of skill, as the players reduce their principal unknowns of internal interdependencies through successive plays to the skill game's major unknown of the optimum strategy. In educational games, there is the progression from process thinking (reality games) to strategic thinking (skill games). Games of reality may also become games of fantasy, if the reality game players become emotionally overcommitted to the game. The same tendency toward fantasy exists in games of chance. Thus it appears that the two stable poles among the four types of games are skill games and fantasies. Put another way, the two stable extremes of games are systems analysis and pure drama. Various degrees of the two can be combined to produce various specific types of educational experiences.

The relatively pure types of games discussed above can each be directed toward achieving specific educational objectives. Games of skill, in which outcomes depend on player capabilities, motivate intellectual achievement and reward individual responsibility and analytic thought. They are best applied in the development of specialized competencies, such as those of mathematical, economic, and social analysis and rational planning. On the other hand, games of pure skill can discourage the slow learner if the players are a group of diverse intellectual capacities. The skillful may become conceited and the less skillful hopeless. Thus in situations where students have a wide range of individual capacities, games of pure skill should not be used. Games involving both skill and some degree of chance are here more desirable.

CHARACTERISTICS OF FOUR TYPES OF GAMES

Events determined and regulated by rules Competition usual Isolated individuals or teams Justice (equality) among players regulated by skill (merit) vs heredity (chance) --similar to Professions Classical--Apollonian Abstraction or specialization	Games of SKILL ← (Economic competition, chess, tennis) (Principal unknown: optimum strategy) ↑	tendency Games of CHANCE (Speculation, dice, roulette) (Principal unknown: Causality) tendency
Regulated by effect achieved (rules determined by desired situation) Cooperation usual Social groups Compensatory (dominant-dependent) hierarchical & traditional relationships --similar to Arts Roamntic--Dionysian Role Play and total involvement	tendency Games of REALITY tendency (Simulation) (Realistic drama, monopoly, war games) (Principal unknown: Interdependencies)	↓ → Games of FANTASY (Exhilaration) (Music, spectacle, dance, skiing) (Principal unknown: Emotional response)
Note: Horizontal and vertical combinations tend to be mutually compatible; diagonal combinations mutually competitive. The stable poles of games activities tend to be skill and fantasy. (Chance degrades into fantasy, reality when understood becomes differentiated into games of skill)	Control Calculation Action EGO-Regulation Analytic Realization Order Strategic (many moves) "Fair" Ethnically creative	Submission Spontaneity Passivity ID/Imagery Inventive Potentiality Freedom Tactical (one move) Arbitrary Culturally creative

TABLE I

Pure games of chance, in which the situation outcomes are independent of player capabilities, tend to erase player inequalities and encourage the slower learners. Anyone can win! Thus it is understandable why gambling enjoys a larger market among the lower socio-economic levels of the population. There is also a certain humbling of the superior minds in games of pure chance that may help them achieve a more realistic view of their own limitations in a complex world, and help them to experience failure despite rational planning in a non-catastrophic way.

One wonders what the effect on Generals Schlieffin and Von Moltke might have been, had their war games been more probabilistic and not wholly determined by skillful, rational planning. Perhaps they would have had a less absolute confidence in the effectiveness of their own good planning, and been more sensitive to the possibilities not included in their war games--such as those things that actually did happen (the British entry into the war after the German invasion of Belguim in 1914) and ruined their plans.

Games of pure chance can discourage rational planning by discounting individual skillful effort, tend to obscure relationships among phenomena, and thus encourage superstition and passivity. They should in their pure form be used very sparingly for education and only to illustrate probability theory itself. However, some probabilistic elements can fruitfully be integrated with educational skill and reality games, to develop a healthy respect for uncertainty, risk, and the dangers of assuming that one can always analyze all the factors in a situation. Intellectual arrogance can thus be educated away to some degree.

Games that model reality are simulations or dramatizations of some adult non-play behavior. They are the most broadly educational form of games because they have the most concrete reality content. Reality games provide information on the facts of the simulated real world situation to the players; they provide dynamic illustrations or working models of the processes simulated; and they give the players opportunities to experiment with alternative strategies for achieving objectives. The players of reality games experience new possibilities beyond their own direct experience. Indeed, the first reality games--war games--were developed precisely for that reason. Experimentation with alternative war plans in reality is impractically costly,, thus the war game as planning and training device.

Games of pure reality are often impractical, because the expense of making and exercising a model of reality with no simplification amounts to reality itself (except for its having been consciously planned). In some cases, such as large-scale military maneuvers, the cost is justified by the education achieved, but even here realism is far from complete. (Soldiers are not intentionally killed). The impracticality of perfect realism requires secondary means for achieving overall problem realism, such as the use of probabilistic outcomes for supporting phenomena.

Pure realism is impossible in games that are still games, but even nearly pure realism may also be undesirable for education purposes at least. Players may become overconfident that the too realistic game is "just like reality", and over-rate the significance of the outcomes. Spurious analogies between game plays and real situations may be made, and the wrong lessons may be learned too well. Thus in educational model reality games it is very important to emphasize the non-predictive, limited degree of the simulation of reality. Boundaries between the reality game and actual reality must be made and kept clear.

Psychological alienation from reality games may also occur, if emotional involvement becomes too great as a result of a failure to keep in mind the distinction between game and reality. This "boundary failure" can also result in obsessional play.

Reality games are probably the most effective form of educational games if the above caveats are observed. If incorporating both skill and chance, they can provide the best balance of motivation, information, comprehension, and social interactions.

Games of fantasy or exhilaration are often not considered games at all, because they are often not competitive or of uncertain outcome. Nevertheless, they are certainly a form of play, and can also be highly educational. Typical fantasy games or play activities are dancing, skiing, singing, fairy tale telling. These fantasy games can refresh the imagination and integrate emotion with cognition. In their pure form, fantasy games are not directly highly productive educationally, but may offer the emotional release between more intellectual activities. In its less restrained form, fantasy can reduce respect for rules, danger, and analysis, but this is readily avoided by common social discipline.

Table II summarizes some of the educational advantages and disadvantages of the four basic types of games described, and gives an opinion on their best educational application.

TABLE II

Advantages-Disadvantages of Educational Games

Type of Game	Educational Advantages	Educational Disadvantages	Best Educational Application
SKILL (situation outcome dependent on capabilities) Examples: Business, Chess Tennis)	recognizes and rewards achievement (effort + skill), discourages laziness, encourages individual responsibility self-control	discourages the backward, dramatizes natural inequalities, feeds the conceit of the skillful. Corrupted form: tyrannical superiority overweening ambition, ruthlessness	motivate and develop specialized competence, self-reliance, rational planning
CHANCE (situation outcome independent of player capabilities) (Examples: Dice speculation, romance, roulette, horse racing)	recognition of personal limitations on effort and skill, erases natural inequalities, humbles overachievers, encourages underachievers	erases personal responsibility, discounts individual effort and skill, encourages magical thinking, denies relationships among phenomena, encourages passivity Corrupted form: gambling, superstition, addiction, fatalism, despair of own capabilities	develop capacity to deal with failure, respect for uncertainty, understanding of probability and risk
MODEL REALITY (simulation of non-play behavior)* (Examples: Fiction, Art, Monopoly, Military maneuvers, Theater)	understanding of problems, motives, and methods of others, experience of new possibilities beyond own direct experience, exploits love of adult reality	spurious analogies, over-rating of predictiveness, alienation if intensity too great, boundaries between game and reality confused Corrupted form: Obsession, delusion	motivate and develop generalized competence, comprehension of context, cooperative social behavior
FANTASY EXHILARATION (release from conventional perception and inhibitions) (Examples: Dancing, Skiing)	creative vision, emotional refreshment, stimulation of imagination	low cognitive content, short endurance of exhilaration activities, destroys respect for rules, physical danger Corrupted form: Drug addiction Alcholism	provide recreation, rejuvenation of interest, emotional catharsis, release from rigidity

Calculation/Spontaneity, competition/cooperation are varied in all types, may be with self, nature or other players.

Showdown (autonomous competition) or strategic games (interacting competition) may occur in all types.

* "Play is a dramatization of adult activity"--Herbert Spencer.

MATH LAB KIT: #11

VENN RINGS AND DIAGRAMS

Materials for Making Venn Rings:

Sources for making Venn-type diagrams or rings include:

1. Solid core insulated wire: Such wire has didferent colored insulation covering the solid core wire. Sources of such wire are Romex house wiring (comes in three strands--red, white, black), telephone cable wiring, and other insulated cables. These can be connected together in several durable ways: crimping, soldering, twisting, or by using electrical wiring fasteners. To prevent sharp wire ends, small amounts of plastic electrical tape can be wound around the connections.

   Small 18" lengths are usually long enough to get a Venn ring with 9"-12" diameters. If larger rings are desired, other materials listed under #2 will serve better.

2. Colored, hollow plastic tubing: Tubing can be connected by inserting an inch or less length of dowel into the loose ends and then taping the connection. Large Venn rings can be made from "Hoola-Hoops." These are especially appropriate if large objects are being classified.

3. Lengths of colored yarns: Especially yarns which are large strands are useful. These can be tied into the sizes of rings desired. These have the disadvantage over the other types described above because they lack rigidness.

4. Overhead projection Venns: Overhead transparencies can be marked with large circles and "overlayed" to gain the Venn diagrams desired. Small objects of different shapes will cast shadows when projected.

5. Diagrams of Venn rings: Ditto sheets with different arrangements of two, three or more Venn rings in various arrangements can be useful with some problem situations.

6. Graph paper: Use different geometric shapes (polygons, etc.) to form rings for representing different operations and relationships.

7. Egg cartons (dozen) or partitioned boxes: Any container which has partitioned sections serve the same function as Venn rings and have the advantage of being able to hold solid manipulative objects.

Ideas for using Venns in learning of Math concepts can be found in such sources as the following:

1. State Math Guide.
2. Handout in Module CMM:006:09 (ISU), "Venn Diagrams Strengthen Children's Mathematical Understandings."
3. Dumas: Math Activities for Child Involvement.

4. Kidd: The Laboratory Approach to Mathematics
5. Operations on Sets, Relationships between and among sets.
    - Collections
    - Union (addition and subtraction)
    - Intersection
    - Disjoint
    - One-to-one Correspondence
    - One-to-many Correspondence
    - Subsets
    - Cardinal numbers of sets
    - Commutative, associative and distributive properties
    - Equivalent and non-equivalent sets
    - Empty or void sets
    - Set partitioning (division)
    - Concept of multiplication

Attribute Work With Venn's:

Using collections of many different types of small manipulative items, learners can create many game-like activities by soring objects by attributes utilizing the Venn rings to organize their data. Charting of such data should be encouraged.

Examples of objects:

- beans
- corn
- small plastic toys (animals, people....)
- buttons
- shapes (colored) made from felt or heavy cardboard
- nuts, bolts, screws, nails, etc. (hardware items)
- paper clips (different sizes, shapes.....)
- marbles
- counters (tongue depressors....)
- colored beads (size, shapes, colors....)
- ETC....

NOTE: Interns are urged to involve learners in making collections of such small objects. These can easily be stored in small boxes (plastic, cardboard or wooden), or in small cloth pull-string bags which learners can easily make from scraps of heavy material such as denim (or their mothers can make).

Storage is an important consideration when setting up Math Lab environments. Items need to be easily stored, easily returned to storage at clean-up time, and easily marked for quick indetification.

Refrigeration containers are excellent because they have snap-seal lids.

Math Activity Cards and Guidesheets:

The Intern is encouraged to develop (and have learners develop) Math Activity Cards and/or Guidesheets for concepts and problems which can utilize Venn diagrams and rings. Some examples:

1. Give the learner(s) a set of three to six Venn rings and a box of mixed objects which vary by color, shape size, use, etc.

   Initiate learner activity with an Activity Card or by suggesting that the learner remove five objects (at random) from the box and arrange them into groups using the Venn rings. DO NOT suggest how the Venn rings should be arranged; allow the learner to explore his groupings. DON'T suggest how the objects should be grouped. Teachers often "rob" learners of thinking or trying and then testing the goodness of their own ideas with their own experiences.

2. Develop Acitivity Cards or Guidesheets that pose problems with sets, subsets, operations on sets, etc. and have the learners utilize Venn rings and/ or diagrams to explore the problems. Examples of Activity Cards or Guidesheets which could lend themselves to such explorations are:

   Activity Cards: Copies in CMM:006:05 (ISU) Module

AS-1	"Addition and Subtraction"
S-1	"Set Relations"
S-2	"Equivalence Relations"
AP-1	"Commutative Property"

   Guidesheets: See Kidd

AC-1	"Attribute Cards" (pp. 77-79)
RB-1	"Ratios" (pp. 230-231)
RA-4	"Ratios" (pp. 217-218)

3. Pose various problems for learners to explore which involves using Venns outside the classroom. For instance, an ecology-type science activity which can effectively utilize Venns is....

   Using three "Hoola-Hoops" or large Venn rings, explore what you find in three different locations---grassy school yard, a grassy, side-of-the road ditch and an unplowed, grassy pasture. The learners can sample such items as grasses, soils, rocks, etc. within the hoop and chart their results.

4. Encourage learners to explain a problem involving NO WORDS, by using only the visual-manipulative means to represent the problem. In such experiments the Teacher and other learners should involve themselves without words also when they pose questions or want to make suggestions.

MATH ACTIVITY CARDS*

INTRODUCTION

Important changes are taking place in elementary school mathematics, and fresh new ideas are being introduced. But while we accept new ideas, critically, we must not overlook old ones that have worked well in the past. Over half a century ago, in 1916, the philosopher Alfred North Whitehead, in his presidential address to the Mathematical Association of England, said: "In training a child to activity of thought, above all things we must beware of what I will call 'inert ideas'--that is to say, ideas that are merely received into the mind without being utilized, or tested, or thrown into fresh combinations.... Let the main ideas which are introduced into a child's education be few and important, and let them be thrown into every combination possible." In the frantic pace of modern education, we run the risk of throwing too many ideas, some perhaps of minor importance, in too short a time, at children who are not always ready to receive them.

Of all the subjects taught in elementary school, mathematics is the most abstract and, in the present as well as in the past, the most likely to produce "inert ideas" in children who are exposed too rapidly to its abstractions. "I was never very good at math" is a statement one hears frequently from adults. Perhaps you have said it yourself. Could it be that you too are overloaded with inert ideas and are inexperienced throwing the mathematical ideas you have learned into new combinations?

There is an ancient Chinese proverb which goes:

> I hear and I forget.
> I see and I remember.
> I do and I understand.

We do not really understand the inert ideas with which our minds are cluttered because we have not done enough with them; we have neither used them nor thrown them into fresh combinations. Thus, a child may learn that there are 12 inches in 1 foot, but this idea will remain inert until he actually begins to measure lengths.

For the past forty years, important research has been carried out by Professor Jean Piaget and his colleagues in Geneva, Switzerland. This research has often focused on the stages through which children must pass as they learn basic mathematical concepts. A main result of this research has been the discovery that children between the ages of five and eight are passing from the stage of "intuitive thinking" to what Piaget calls the stage of "concrete operations." The adult warning, "Things are seldom what they seem" applies profoundly to a child in the "intuitive stage." For example, in one of Piaget's most famous experiments, water is poured from a short, fat glass into a tall, thin one in front of the child. It looks like there is more water in the tall glass, and the child still in the intuitive stage really believes there is. Because it looks like more water, it is more water. Later, when a child passes into the stage of concrete operations, he will understand that the water looks like more in the tall glass. If he poured it back into the short, fat glass, and it looked like less, it wouldn't really be less.

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*Adapted from: Teacher's Guide, the Macmillan Math Activity Cards; by David M. Clarkson, Macmillan Co., 1970. Pp. 1-9.

The research of Piaget has shown elementary school educators that many basic concepts, so obvious to adults that they aren't even noticed, are not yet part of the thinking of young children. This discovery has profound consequences for elementary mathematics education. First, it means that we must learn to diagnose the stage our pupils have reached, and then we must learn to adjust our curricula to fit that stage. Some Math Activity Cards have been designed for use by seven and eight-year-old children who have just moved out of the stage of intuitive thinking into the stage of concrete operations. However, there are likely to be some children in second and third grade classes who are still in the intuitive stage. Special provisions should be made for these intuitive thinkers.

Most second and third grade children have reached the stage of concrete operations, but some of their thinking will be intuitive for many years. Occasional backsliding into an earlier stage is more the rule than the exception. However, the leap forward to the third Piaget stage--"formal operations," or logical thought-is not at all uncommon. For example, it would be foolish to teach concepts of area to children in the intuitive stage because a shape would be as big as it seems to them--a change in perspective would mean a change in size. A child in the stage of concrete operations, however, is ready to explore the concept of area. By working with physical units of area, such as pieces of cardboard, he can find out how many pieces are needed to cover a surface. A strong hint that a child is passing into the stage of formal, or logical, thinking is when you see him reject the physical pieces of cardboard and note that "you can do it by multiplying." Of course, if someone has told him that he will find areas by multiplying, this may be just one more inert idea; but if he has found it himself, then it is an important developmental sign.

Math Activity Cards are written both for those children in the stage of concrete operations who occasionally may slide back into intuitive thinking or jump ahead into formal, logical thought. Cards may deal with such basic topics as: graphs, shapes, measurements, patterns, and mathematical games. They can be scaled to levels of maturity and emphasize manipulation of physical objects appropriate to the stage of concrete operations. The levels of the Math Activity Cards build up certain basic mathematical skills recognized as important in modern mathematics programs: graphing various functions with the use of Cartesian coordinates, recognizing and classifying shapes and relations of shapes, making measurements of all sorts with varying degrees of accuracy, developing an eye for patterns in mathematical situations and using the patterns to simplify tasks and deepen understanding. The cards can be designed to take about one-fifth to one-fourth of the time normally devoted to elementary mathematics instruction. They may be used with any contemporary elementary mathematics textbook series.

WHY CARDS?

Until recently, much elementary mathematics instruction has relied upon standardized textbook series. Teachers would explain an arithmetical process, and the children would then complete exercises in the textbook that were designed to provide the practice necessary for retention. The chief advantage of texts is that they provide a control over curricular development; the teacher knows roughly what gound has been covered with the class. The main disadvantage of using textbooks is that rarely are all children in a classroom ready for the same material at the same time. Another disadvantage is that a too heavy dependence on textbook learning promotes a passive role for the children at their desks. Piaget's research, and that of may others, has shown that children of elementary school age need active learning situations that get them up out of their chairs and into problem-solving tasks involving physical activities. Of course, experienced teachers have always provided these active experiences for their pupils, but they have left the traditional textbook lesson to do so.

The main advantage of mathematics activity cards is that they provide, within the basic curricular area of the grade level, a means for children to select tasks that are appropriate to their own level of development. The children may then pursue these activities in a laboratory classroom setting. Pupils may work through the basic set of cards at their own pace. They may defer work on a given card until they are ready for it. They may spend varying lengths of time on cards because no one is waiting for them to "finish up today's assignment." The cards may be used with this flexibility because they are not sequential within a given level. In working with these cards, there is no need to progress from card 3 to card 4 as there is in a textbook to progress from page 3 to page 4. Furthermore, new cards may be added to the set to meet special needs. These new cards may come from earlier or later levels in the series or, better still, they may be written by the teacher and the children and so be more directly relevant to the local class and school situation. Indeed, the cards in each level should be suggestive rather than requirements. They are written to stimulate thought and action, not to lay down a fixed curriculum. They are not so much to be "worked through" as they are to be "worked with."

WHO SHOULD USE THE CARDS?

The use of Math Activity Cards will vary from class to class and from student to student. The cards are intended for all levels, and some teachers may find it appropriate to use some cards at higher or lower grade levels than those indicated. This flexibility is intentional and contributes to the efficiency of the cards.

OPEN-ENDEDNESS

Most Activity Cards should be open-ended. This means that little specific instruction is given by the cards themselves on how to do the work. Children are expected and encouraged to devise their own methods of solution or exploration. (In fact, they do this very well.) There are profound educational reasons for this open-endedness. For one thing, most of the important questions we face as adults are open-ended. Thus, by providing children with similar questions, we help prepare them for adult responsibilities. But even more important, children learn in a multitude of different ways and rarely in precisely the way we expect them to learn. By giving them open-ended questions and by emphasizing their own inventive methods of solution, we make use of the full force of their learning power. In many of the cards the "answer" should be a discussion between child and child or between teacher and child--a discussion in which both parties talk as colleagues and equals, all sharing a common discovery. Such discussions give a teacher that unique and rare opportunity to get to know a child in a more intimate way than normal classroom procedures permit.

HOW DO YOU GET STARTED?

There are many ways to start using Math Activity Cards. The method you choose will probably be decided by the degree of freedom you have been accustomed to allow your pupils in and about the classroom. If children do most of their classwork at their desks, if the school situation is formal, or if there are crowded conditions, it is wise to start using cards devoted to one topic, say graphing, and to establish "experience stations" around the classroom. In such a case, first lessons with cards might be to hand them out to the children and conduct a class discussion about what you plan to do with them"next week." You might even take one particular card and draw a replica of it on the front chalkboard. Then you could discuss with the class what the card might entail, what equipment the children would need to work with, what alternative methods could be used, what questions might be left unanswered, and how the children might record their discoveries. Finally, this discussion might be necessary as a reading lesson, simply to find out how well the children can read the instructions on a card.

Children who have not been accustomed to freedom of movement in a classroom are likely to be confused by the informality of a laboratory-workshop period. It is therefore most important to plan the first introduction of work with cards in a classroom very carefully. A truly free spirit of experimentation and inquiry can be developed in any elementary school classroom if the proper sequence of orderly steps toward this spirit is followed. The first and most important step is discussion with the children about what will be expected of them in the way of behavior. The motivation provided by the cards should be so strong that few problems arise once the children work properly with them. But it is most unlikely, especially with younger children, that an atmosphere of free, responsible inquiry and cooperation will prevail the first day cards are used! Even with older children it takes time to develop this.

Some of the things you need to discuss with the children are: (1) where they may work, (2) with whom they may work; (3) where to get materials they need; (4) how they must clean up afterwards; (5) how much time they may have; (6) how to keep records of what they learn; (7) how to ask for the teacher's attention; (8) what alternatives there are to working on cards; (9) when they may choose a new card; (10) when and under what conditions they may leave the classroom. Although some of these questions may not arise, and you may not want them to arise, you should be prepared to deal with them in class discussion.

One way to start, particularly if your classroom or school budget makes it difficult for you to purchase equipment, is with an equipment building project. The children can bring boxes and bottles and other needed items from home. Involving the children this way, before they actually work with cards, creates considerable anticipation and a good climate for discussing procedures. Fathers may be enlisted to help make some pieces of equipment. In this way you will develop parent interest in the work with cards. Such informed interest can be of tremendous help to a classroom teacher.

When equipment has been collected and class discussions about procedures has occurred, perhaps one good way to get started is by setting up experience stations about the classroom and dealing with a single topic using six or eight different cards. You might rearrange the desks in your classroom to form six or more clusters, or work stations. If children are unaccustomed to rearranging desks and tables in the classroom, preliminary discussion and planning are essential to avoid confusion. The easiest way to make an experience station is to push some desks together or place some chairs around a table; but variety is a good idea, since desks in a circle or in pairs may be better for some activities. If storage closets or shelves in your classroom can be moved easily, these can be placed at right angles to the walls to make quiet alcoves.

Once your classroom has been rearranged for a lesson with experience stations, each station should be matched with a card, a group of children, and as many of the needed materials as can be anticipated. During the lesson, children may move from one experience station to another as they complete tasks suggested by the cards. Materials produced may be displayed on class bulletin boards; after several lessons of this type, general class discussions may be held on what methods worked and what changes might improve lessons. Children may share discoveries, ask questions, and plan for future lessons. After everyone has had his say, you may decide to grant more freedom during the lessons with cards by allowing pairs or small groups of children to select their cards. Or, you may decide to have more sessions with experience stations. You may even wish to restrict the work with cards to a part of your class while the rest of the children do something else. In this case, you might choose the more mature ones, who are best able to work by themselves, to work with the cards. As others become ready to assume the responsibility, they may be added to the groups working with the cards.

Creating a good classroom atmosphere for working with cards depends on careful planning, adequate discussion with the children about what is expected of them, and wise decisions about making changes once you are under way. No single formula will work, and, as in all good teaching, you should "play it by ear" until both you and the children are well adjusted to the changes.

GROUPING

Different teachers have had varying experiences with grouping. Here are some possibilities with their advantages and disadvantages: You may choose to have the children work in pairs or in larger groups. With larger groups there are fewer different activities to keep track of, but weaker students tend to become passive. Many teachers have found that, for some cards, groups of three are best; however, for most cards the work probably should be done by pairs of students. Occasionally, cards require groups of at least four children because the task is so large. Sometimes, at a particular point, a discussion with the entire class is the best way to handle the work of a card or set of cards; but this is the exception to the rule. Since the cards are designed to be learned from rather than "taught," class discussion of a card should be restricted to the first introductions and to final summings up.

Choosing which children should work together is by far the most sensitive task of the teacher in establishing good working habits and learning experiences with the cards. Pairs or small groups may be based on level of maturity, friendship, or other considerations. If a bright child is paired with a slow one, the pair should be watched to see if the bright child is tolerant of the slow one and if the slow one is doing his share of the work. Pairing based on friendship can be extremely productive but must be watched. It is important to make sure that paired children know under what conditions they are working together and that groupings may be changed. The role of the teacher in establishing these pairs and in evaluating their effectiveness is crucial. On rare occasions it may be wise for a child to work alone for awhile; but as most of the cards promote discussion between children, this is exceptional.

RECORDING

There are two types of records: children's and teacher's. The kinds of records that children keep of their work vary with the tasks, their ages, and abilities. A great deal of the most valuable work done with cards is in the form of a discussion between children, and no record can be kept of that. In many cases a child has considerable latitude in choosing the form of the record he makes; this choice is a vital part of his work. In general, it is unwise to require that a child keep a record of completed work until he is ready. He is ready when he begins to keep records on his own. These first attempts at record-keeping may not look like much to an adult, but they are of vital significance to the child who makes and proudly displays them for all to see. Records of the work of seven and eight year olds are best kept in individual folders throughout the school year. Some records should, of course, be displayed on bulletin boards or "math tables." An overemphasis on recording at the outset can be stultifying and it detracts from the value of the work with cards. By the same token, it would be a great mistake to "grade" recorded work in any conventional sense, such as applying A's or B's to it. The freedom with which children explore new, open-ended materials would be severely curtailed by such a practice.

Teacher's records of work done by children with Math Activity Cards can be of several types. First and foremost are the records a teacher keeps of what she observes while the child is at work in the classroom. One important benefit of work with cards is that the teacher can spend much more time observing individual children than is possible under normal classroom conditions. Anecdotal records of observations of children at work may be placed in the child's folder along with samples of his ordinary laboratory work. A comparison of these records at the year's end may make possible a very accurate appraisal of progress.

Secondly, a teacher may keep a record of cards that have been satisfactorily completed, with the dates on which they were finished. The records of cards completed may be kept as a chart or in a notebook for the class as a whole. Thirdly, a teacher may wish to construct test items for individual children to help evaluate their progress with the cards. Such individual testing may be of particular value in determining the readiness of a child for work at the next level of difficulty. However, it would be a mistake for a teacher to make up a class test based on the cards because each child should have a unique experience working with the material, and class comparisons would tend to defeat the purpose of the cards.

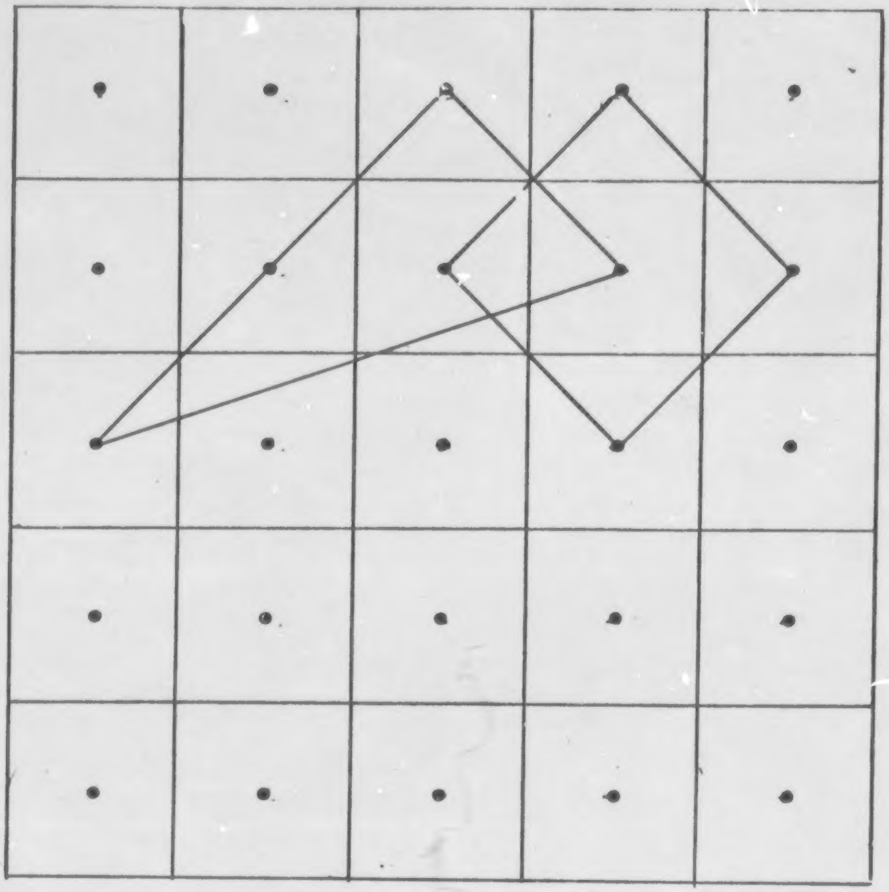
Finally, in discussing record keeping, it is well to remember that you can overdo recording. Keeping too close track of what a child is learning is a bit like pulling a plant up by the roots to see how it's growing. Enjoy the present productive moment enough to be willing to let it fly away unrecorded. For example, when a child or group makes a lovely model or graph, enjoy it with the class but do not keep it around too long. The work that children do is very personal to them and to their friends, but it may have an inhibiting effect on next year's class to be aware of the work of last year's class. Record keeping is only a means to an end, that is, understanding. It should never dominate in good education.

MATERIALS

If the classroom budget allows, you may be able to purchase many items of equipment. Certain items are really very helpful, and you will want to buy them if you can. For example, a stopwatch is very handy; so is a reasonably good pan balance and a set of weights. If large sheets of squared paper and sets of 1-inch wooden cubes are available in the classroom, you will save time. But, if you are severely restricted in spending school funds for these things, adequate substitutes can be made. A watch with a second hand, a homemade balance and replacements of items may be used instead. The location and preparation of the homemade substitutions can provide vital learning experiences for the entire class. For example, a child can learn much from creating his own set of weights from Plasticine, clay, or plastic bags of sand or dirt. If wooden cubes are not readily available, other materials, such as paper, sugar cubes, etc., may be used. In any case, lack of purchased equipment should not deter any classroom teacher from beginning work with the cards.

One of the most useful items, which may be purchased from several sources (but which is easy to make), is a set of nail boards; a 5 by 5 nail board is quite sufficient. A slightly larger board (up to 10 by 10) is also helpful. Nail boards may be made quite easily from spare scraps of wood that are cut into squares 6 inches or more on the side. Linoleum nails (round heads) may be placed at intersection points or as shown in the figure on the next page. Colored rubber bands are available in the five-and-dime.

Perhaps junior-high-school shop students can make a set of nail boards from old desk tops in their shops. Similarly, parents may become involved; if they help make needed materials, such as trundle wheels, there is a chance that more fathers will turn up at the next PTA meeting!



Storage may present a problem, and careful plans should be made for it. Some schools have set aside special rooms for mathematics laboratories. The disadvantage of such special rooms is that they often do not get as much use as they should; nothing is more desolate than a room full of dusty objects that have not been used for months. Well-organized storage in small boxes in the classroom is best.

If material is purchased for sharing among several classrooms, then some equipment might be stored on a rolling storage cart that can be moved quickly from one class to another, as use demands. Such a cart should have a small locked drawer to store small valuable items like stopwatches.

Prices for activity card equipment vary considerably, so it pays to check prices in several catalogs before ordering. Fifty-foot cloth measuring tapes, vary in price from $1.50 to more than $5.00. Trundle wheels, which "click" as they turn, often come with poor quality clickers that break easily. Stopwatches purchased at low prices from discount houses may not be repairable locally due to a lack of parts.

Although materials may be ordered from a large variety of catalog houses, the two following sources are worth noting. CCM: School Materials Division, 2124 West 82nd Place, Chicago, Illinois 60620. Also the article by Patricia S. Davidson, "An annotated bibliography of suggested manipulative devices" (Arithmetic Teacher, Vol. 15, No. 6, October, 1968) contains valuable specific suggestions, prices, and addresses of suppliers. (See Module: CMM:006:04 (ISU).

It is very important that teachers using the activity cards plan well in advance to acquire the necessary materials, either by purchase or from home-construction sources. It is a good idea to involve the children and their parents in this activity. Much thought must go into the writing of Activity cards so that a minimum of expensive equipment will be necessary. It is possible to start using the cards without buying any equipment, but teachers will find that purchasing certain items is extremely useful. Among these are: a number of 1-inch wooden cubes, at least one pan balance of reasonable accuracy, a trundle wheel, and large sheets of squared paper ($\frac{1}{2}$ inch squares).

MAKING ADDITIONAL CARDS

The cards included provide a solid development of certain basic concepts of elementary mathematics. However, they are not intended to be thought of as a final or complete program. Instead, they should be used as an outline of basic ideas from which a classroom teacher may build other cards that are more closely related to the needs and particular qualities of the class, school and community. Any card that turns out to be too difficult for a group of children should be discarded and replaced by one written for the same purpose, but at a more appropriate level. Additional cards may be necessary for some children to help promote the development of basic understandings needed to pass from one learning stage to another--for example, the intuitive to the concrete operations stage. Some children need a wider stage than others, that is, a larger number of fresh combinations. It is likely that more points of view and more examples will be useful. These should be provided by the classroom teacher, who is in the best possible position to determine how necessary they are.

The best source of ideas for replacing and adding to these cards is observation of the children using them. The children will tell you what they need and will even give you the appropriate language to use. In the final analysis, children are our best authors and critics. Proper use of the Math Activity Cards will enable children to perform this crucial role in our schools. In the end, the best set of cards is that set jointly constructed by the pupils and teachers year by year on the basis of their experience and interests. If use of the cards promotes such a result, then the cards have been successful in your school.

MATH LAB ACTIVITY CARDS--DESIGN AND USE*

Many math learning activities can be carried on by learners independent of teacher-direction. A corner of the room, where learners can manipulate physical materials, will often provide needed experiences for concept development.

Learners individually or in small groups can work independently of the teacher with physical objects using Math Lab Activity Cards.

An Activity Card is simply a 5 X 8 sheet on which a particular activity is suggested. Each card lists the activity under study and the suggested level at which the card is to be used.

Such cards may be purchased from several commercial sources[1], be designed by cooperative writing teams, or be written by an individual teacher. Their usefulness is dependent on the experience and resources of the teacher and the nature of the class which is to use the material.

Biggs and Maclean[2], suggest that every Activity Card incorporate the following principles:

1. The learner must DO SOMETHING: go out and measure, organize tables of facts, and so on......

2. The learners must RECORD THEIR FINDINGS in some way that will communicate their procedures and conclusions.

A MODEL MATH LAB ACTIVITY CARD might look something like this:

MATH LAB ACTIVITY CARD number

Title of Topic Level

A. Activity--What is the learner to DO?

B. Activity--What should the learner DO NEXT?

C. Activity--What is the learner's CULMINATING ACTIVITY?

Because Activity Cards are used to help the teacher promote active learning with children, it is important that the activities posed on the Card be "open-ended", allowing learners to approach a problem in a variety of ways. Kidd, Meyers and Cilley[3] provide an excellent example of such a question in a series of GUIDESHEETS within a unit on ratio:

"How can you measure the speed of a pitched baseball?"

This question provides no guidance from the teacher as to the procedures to be followed by the learners, nor is any unit of measure specified. The learner is free to EXPLORE various means of solving the problem and is thus given the opportunity of DISCOVERING a solution for himself.

To aid a teacher or a trainee in the use of MATH LAB ACTIVITY CARDS, samples of different TOPICS and LEVELS of Activity Cards are attached in a Kit. A TEACHING NOTES section has been provided for each sample Activity Card. These NOTES provide brief suggestions concerning the preparation of materials, etc., and NOTES concerning the purpose, procedures and values of the particular activities. The TEACHING NOTES are only suggestive and a teacher or trainee will want to vary from these ideas with appropriate adjustments. It is urged, however, that any adjustments in the Activity Cards still conform with the principles for Activity Cards stated earlier.

The Levels suggested on the sample Activity Cards are coded as follows:

EP-----Early Primary Level (K-1)
P------Primary Level (1-2)
LP-----Late Primary Level (3)
MLP---Middle-Late Primary Level (2-3)
EI-----Early Intermediate Level (4)
I-------Intermediate Level (4-6)

Other labels could be used to indicate difficult of the activities.

The codings used for the various topics are only suggestive of how to label the topics of the Activity Cards. Whatever system is used should correspond to the Objectives being taught and should allow for a multi-level development of topics in order to support individualization efforts. The Coding System used in the sample Activity Cards are as follows:

Topic			Level
Sets	S-1	Set Relations	EP
	S-2	Equivalence Relations	EP
Addition	A-1	Addition Algorithm	P
	A-2	Using Addition	EI
Addition-Subtraction	AS-1	Addition and Subtraction	EP
	AS-2	Rationalizing Integers	I
Properties of Addition	AP-1	Commutative Property of Addition	EP

Topic			Level
Multiplication	M-1	Introduction to Multiplication	P
	M-2	Beginning Multiplication	P
	M-3	Multiplication Algorithm	I
	M-4	Multiplication of Integers	I
	M-5	Negative Integers	I
	M-6	Multiplying with Exponents(A)	I
	M-7	Multiplying with Exponents (B)	I
	M-8	Multiplying with Exponents (C)	I
Properties of Multiplication	MP-1	Associative Property of Multiplication	P
	MP-2	Distributive Property of Multiplication	LP
	MP-3	Distributive Property of Multiplication	I
Division	D-1	Natural Division Algorithm	I
	D-2	Estimation with Division Algorithm	I
	D-3	Division with Integers (A)	I
	D-4	Division with Integers (B)	I
	D-5	Verifying a New Check (Division)	I
	D-6	Division with Exponents (A)	I
	D-7	Division with Exponents (B)	I
Properties of Division	DP-1	Distributive Property of Division	LP
Rational Numbers	RN-1	Recognition of Fractions	P
	RN-2	Equivalent Fractions	LP
	RN-3	Fractions: Describing a Set	I
	RN-4	Addition of Fractions	I
	RN-5	Multiplication of Fractions (A)	I
	RN-6	Multiplication of Fractions (B)	I
	RN-7	Addition of Mixed Numbers	I
Ratio and Proportion	RP-1	Ratio and Proportion	I
	RP-2	Algorithm for Ratio and Proportion	I
Decimals	DC-1	Decimal Fractions	I
Percentage	P-1	Percent (A)	I
	P-2	Percent (B)	I

Topic			Level
Measurement	M-1	Beginning Measure	MLP
	M-2	Beginning Measure	MP
Graphing	GR-1	Coordinates	L
Mental Arithmetic	MA-1	Mental Arithmetic	EI
Subtraction	SU-1	Subtraction Review	I
Place Value	PV-1	Expnnentiation	I
Numeration Systems	NS-1	Expressing Amounts Through Grouping	I
	NS-2	My Numeration System	I
	NS-3	Egyptian Math Fun	I
	NS-4	Number Base, Groups of 3	I
	NS-5	Non-Decimal Algorithm	I
	NS-6	Exploring Mod 7	I
	NS-7	Mod 3 Addition	I
Geometry	G-1	Classifying Shapes	EP
	G-2	Point	EI
	G-3	Lines, Segments, Rays, Curves	I
	G-4	Working with Rectangles	I
	G-5	Parallelograms	I
	G-6	Working with Circumference	I
	G-7	Finding Areas	I
	G-8	Formula for Rectangular Regions	I
	G-9	Circular Regions	I
	G-10	Volume	I
	G-11	Volume of Cone	I
Time	T-1	Awareness of Time	P
	T-2	Telling Time	P
Number Theory	NT-1	Permulations	I
	NT-2	Number Theory	I
	NT-3	Primes	I
	NT-4	Probability	EI

NOTE: No sequence is implied in the numbering of the above Activity Cards; however, Cards should be sequenced as levels of a skill are developed .

REFERENCES

* Adaptions of Activity Cards from C. W. Schminke, Norbert Maertens and William Arnold; Teaching the Child Mathematics; Dryden Press, 1973.

1. Sources for Activity Cards:
   a. Kidd, Meyers and Cilley: The Laboratory Approach to Mathematics (see for examples of GUIDESHEETS.)
   b. See bibliography attached to Module CMM:006:04 (ISU) for many examples of materials utilizing "Activity Cards."
   c. Math Applications Kit, by Allen C. Friebel, Carolyn Kay Gingrich, Science Research Associates, 1971.
   d. Math Activity Cards, The Macmillan Co., 1970.
2. Edith Biggs and James Maclean: Freedom to Learn, Addison-Wesley Ltd., Canada, 1969, pp. 14-16.
3. Kenneth Kidd, Shirley Meyers and David Cilley: The Laboratory Approach to Mathematics, Science Research Associates, 1970, p. 254.

TEACHING NOTES: MATH LAB ACTIVITY CARD

S-1 Set Relations (EP) Early Primary Level

Preparation:

Arrange on a table in random fashion such objects as the following:
cubes spheres pencils discs crayons, etc.

About 12 objects is sufficient, but limit the number of different objects to 3 or 4. The objects may be of different color, size, material make-up, or other attributes. This variety is necessary in order for the learners to respond to ITEMS D and E on the Activity Card which asks learners to re-arrange the objects in different ways based upon different attributes.

NOTES:

Children using the card are actively involved in manipulating objects and exploring relationships. A meaningful review of set properties and one-to-one correspondence is included, along with work to develop concepts of equivalence of sets and of comparison of sets by their numerousness. This notion, the idea of "sameness" of collections, must be examined by physical pairings so that it becomes perceptually clear when collections are alike so far as their numbers are concerned.

Since such an Activity Card as S-1 is meant for "Early Primary Level" learners, it may require reading skill NOT YET possessed by learners at this level. It will be necessary, therefore, that either an older learner (such as a child from grade 4, 5 or 6), or an adult (such as a Teacher Aide, a

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MATH LAB ACTIVITY CARD S-1 (EP)

Set Relations

A. Put the objects on the table into groups that belong together.

B. Are there any sets having the SAME number of objects? PLACE the objects side by side like this:

C. Which sets have the MOST objects?
Which sets have the LEAST objects?

D. Is there any other way you could ARRANGE the objects so that they belong together?
Now, which sets have the SAME number of objects? PLACE these objects side by side.
Which sets have the MOST number of objects?
Which sets have the LEAST number of objects?

E. See in how many ways you can GROUP objects that belong together.
Find the sets that have the SAME number of objects.
Which have the MOST objects?
Which have the LEAST objects?

TEACHING NOTES: MATH LAB ACTIVITY CARD

S-1 Set Relations (continued)........

Parent Volunteer, a Teacher Trainee, or even the teacher) give assistance in reading the instructions on the Card. To allow the learners to get actively involved with the Card and the objects, this assistance should be limited to "reading the instructions."

S-2 Equivalence Relations (EP) Early Primary Level

Preparation:

Arrange on a table in like groups such objects as those used in S-1 Activity Card.

One set of objects should have a distinctive characteristic, such as color (RED) to serve as the comparison set. Others sets would then be examined in relation to the comparison set to determine the various equivalence relationships, such as "sameness," "fewer," "more," "most," etc.

NOTES:

Using examples such as these, learners are encouraged to discover for themselves that one basis for comparing groups is numerousness. Such comparisons are essential for complete understanding of cardinality, as children learn that such divergent sets as apples and baseball gloves can have one property in common and that common peoperty is numerousness. Thus, numerousness is that aspect of number which expresses the quantitative state of a collection and is generally referred to as cardinal number.

MATH LAB ACTIVITY CARD S-2
(EP)

Equivalence Realtions

A. Find those sets on the table which have the same number of objects as the set of RED blocks.

Which are they?

B. Find the sets which have FEWER members than the set of red blocks.

Which are they?

C. Find the sets which have MORE members than the set of red blocks.

Which are they?

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TEACHING NOTES: MATH LAB ACTIVITY CARD

S - 1 Set Relations (EP) Early Primary Level

PREPARATION:

Arrange on a table in random fashion such objects as the following:

cubes spheres pencils discs crayons, etc.

About 12 objects is sufficient, but limit the number of different objects to 3 or 4. The objects may be of different color, size, material make-up, or other attributes. This variety is necessary in order for the learners to respond to ITEMS D and E on the Activity Card which asks learners to re-arrange the objects in different ways based upon different attributes.

NOTES:

Children using the card are actively involved in manipulating objects and exploring relationships. A meaningful review of set properties and one-to-one correspondence is included, along with work to develop concepts of equivalence of sets and of comparison of sets by their numerousness. This notion, the idea of "sameness" of collections, must be examined by physical pairings so that it becomes perceptually clear when collections are alike so far as their numbers are concerned.

Since such an Activity Card as S-1 is meant for "Early Primary Level" learners, it may require reading skill NOT YET possessed by learners at this level. It will be necessary, therefore, that either an older learner (such as a child from grade 4, 5, or 6), or an adult (such as a Teacher Aide, a Parent Volunteer, a Teacher Trainee, or even the Teacher) give assistance in reading the instructions on the Card. To allow the learners to get actively involved with the Card and the objects, this assistance should be limited to "reading the instructions."

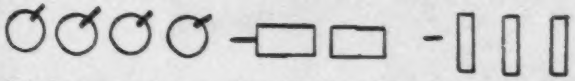
MATH LAB ACTIVITY CARD S-1

Set Relations (EP)

1. Put the objects on the table into groups that belong together.

2. Are there any sets having the SAME number of objects?

   PLACE the objects side by side like this:



3. Which sets have the MOST objects?

   Which sets have the LEAST objects?

4. Is there any other way you could ARRANGE the objects so that they belong together? Now, which sets have the SAME number of objects? PLACE these objects side by side.

   Which sets have the MOST number of objects?

   Which sets have the LEAST number of objects?

5. See in how many ways you can GROUP objects that belong together.

   Find the sets that have the SAME number of objects.

   Which have the MOST objects?

   Which have the LEAST objects?

CMM:006:05 (ISU)

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TEACHING NOTES: MATH LAB ACTIVITY CARDS

S-1 Intersection of Sets (P) Primary Level

Preparation

5 X 5 Geo-Board
Colored rubber bands

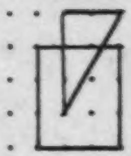
NOTES:

Do Not be concerned if some learner says, "Which triangle, the little one or the big one?" He can decide for himself in terms of what the entire Lab Card asks him to do.

This card will involve the idea of subsets, universe, sets, but no attempt to label them or define them should be made by the teacher.

MATH LAB ACTIVITY CARD S-1 (P)

Intersection of Sets



1. How many pegs are inside the triangle?
2. How many pegs are inside the square?
3. How many are common to both the triangle and the square?
4. How many pegs are in neither the square nor the triangle?
5. How many pegs are in the square but NOT in the triangle?

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TEACHING NOTES: MATH LAB ACTIVITY CARDS

S-2 Intersection of Sets (P) Primary Level

Preparation

5 X 5 Geo-Board
colored rubber bands

NOTES:

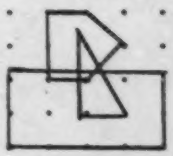
This kind of card can well be a problem for some learners who need to learn to follow directions.

Use of different colored rubber bands will help the learner see these relationships. This exercise is very close to Venn ring or diagram work and could be used in conjunction with Venn Rings.

MATH LAB ACTIVITY CARD

S-2
(P)

Intersection of Sets



1. How many pegs are within the triangle?
2. How mamy pegs within the 5-sided figure but NOT within the triangle?
3. How many pegs within the triangle but NOT within the rectangle?
4. How many pegs are common to both the triangle and the rectangle?
5. How many pegs are common to all three figures?
6. How many pegs are NOT in any of the three figures?

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TEACHING NOTES: MATH LAB ACTIVITY CARD

S-2 Equivalence Relations (EP) Early Primary Level

PREPARATION:

Arrange on a table in like groups such objects as those used in S-1 Activity Card.

One set of objects should have a distinctive characteristic, such as color (RED) to serve as the comparison set. Other sets would then be examined in relation to the comparison set to determine the various equivalence relationships, such as "sameness," "fewer," "more," "most," etc.

NOTES:

Using examples such as these, learners are encouraged to discover for themselves that one basis for comparing groups is numerousness. Such comparisons are essential for complete understanding of cardinality, as children learn that such divergent sets as apples and baseball gloves can have one property in common and that common property is numerousness. Thus, numerousness is that aspect of number which expresses the quantitative state of a collection and is generally referred to as cardinal number.

MATH LAB ACTIVITY CARD S-2

Equivalence Relations (EP)

1. Find those sets on the table which have the same number of objects as the set of RED blocks.

   Which are they?

2. Find the sets which have FEWER members than the set of red blocks.

   Which are they?

3. Find the sets which have MORE members than the set of red blocks.

   Which are they?

TEACHING NOTES: MATH LAB ACTIVITY CARDS

A-1 Addition Algorithm (P) Primary Level

Preparation:

Have quantities of concrete materials available for utilizing during problem solving experiences.

NOTES:

Merely presenting an algorithm to learners without them seeing a need for it is not helpful. This provides them no basis for determining the utility of the algorithm except on the basis of the teacher's explanations.

Learners need to develop and use algorithms thru experiences in problem solving situations, utilizing concrete materials. Such problems as the following, using concrete experiences, provide representations for the actions described by an algorithm:

> "A boy has 12 blocks and goes to the shelf and gets 15 more. How many blocks does he have in all."

Learners will obtain solutinns to such quantitative situations by whatever means that seem most appropriate; most solutions requiring concrete manipulations. But learners will soon seek ways to ease their work by creating shortcuts. At such a time it is appropriate to examine algorithms with learners systematically. An Acitivity Card such as A-1 can help learners discover and explore addition algorithms.

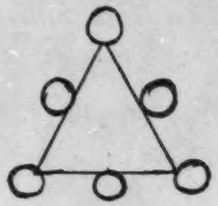
Discovery of the need for, and the subsequent development of, a math algorithm requires the following:

MATH LAB ACTIVITY CARD

A-1 (P)

Addition Algorithm

1. Complete the magic triangle using:
   - (a) One digit numerals (1, 2, 3, 4, 5, 6, 7, 8, 9)
   - (b) No numeral more than once so all sides equal 17.



2. Supply the missing digits in each table:

a.

+	3		
		2	
	7		12

b.

+	3	3		7
	8	9		2
		4		
			6	5
	4	2		9

3. Fill in the missing digits in the additions below:

a. $\begin{array}{r} 3 \\ +\ 7 \\ \hline 63 \end{array}$

b. $\begin{array}{r} 6 \\ +9 \\ \hline 134 \end{array}$

c. $\begin{array}{r} 5 \\ +5 \\ \hline 88 \end{array}$

Stimulation by a quantitative problem.
Freedom to explore, manipulate, and
record ideas.

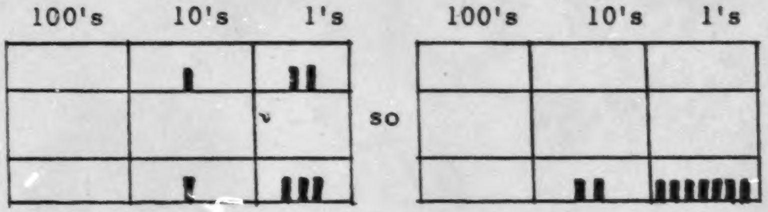
Of course, early attempts by learners at forming algorithms are likely to be inefficient. They should be encouraged to use, and refine, their algorithms; this is the first step in moving from concrete to the abstract.

Learners who develop their own algorithms clearly relate them to the problem situations which exist! Their algorithms are unique but should be compared in order to move them toward more efficient algorithms that may be used.

Vertical and horizontal formats should be explored. Emphasize the number action through some concrete aid (pocket charts, markers, counters, etc.). Review the number action at the same time they are studying the algorithm.

Number Action---"combining"

$$\begin{array}{r} 12 \\ +15 \\ \hline \end{array} \qquad 12 + 15 = \begin{aligned} &= (10 + 2) + (10 + 5) \\ &= (10 + 10) + (2 + 5) \\ &= 20 + 7 \\ &= 27 \end{aligned}$$



This kind of process allows the learner to combine

the two-10's first and then the two-1's and three-1's.

As they gain confidence with this manipulation, they will easily move toward the traditional algorithm whereby the ones are added first and the tens next.

The horizontal algorithm as an initial algorithm, has the advantage of graphically showing the "numbers action" (combining or regrouping) which addition requires.

Three digit additions encourage considerable practice of both the commutative and associative properties. (Learners will note that any of the digits may be added first, as long as column value is not violated.)

$$\begin{aligned} 634 + 153 &= (600 + 30 + 4) + (100 + 50 + 3) \\ &= (600 + 100) + (30 + 50) + (4 + 3) \\ &= 700 + 80 + 7 \\ &= 700 + (80 + 7) \\ &= 700 + 87 \\ &= 787 \end{aligned}$$

Such algorithms illustrate patterns that accommodate computation--the reason for an algorithm. After all, an algorithm is merely a convenience that permits us to record our thinking! It has no special meaning inherent in the algorithm itself!

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TEACHING NOTES: MATH LAB ACTIVITY CARD

A-2 Using Addition (EI) Early Intermediate

PREPARATION:

Send a group or individual learners dealing with ACTIVITY CARD A-2 to an area where musical instruments are stored.

NOTES:

There are a number of activities which help learner USE addition.

$$\left.\begin{array}{l}\text{describing}\\ \text{classifying}\\ \text{recording}\end{array}\right\}\ \text{quantitative data}$$

Data in classroom, inside and outside of the school buildings, and playgrounds are rich sources for math learning, especially uses of addition.

Types and number of shrubs and trees.
Types and numbers of furniture, building materials,
 play areas or equipment, books.

Many attributes for classifying objects exists--content, color, size, shape, length, etc. and adding such groups differs useful addition practice.

MATH LAB ACTIVITY CARD A-2

Using Addition (EI)

1. Describe and count the musical instruments you find.

2. Arrange your descriptions in sets.

   How many sets did you find?
   How many instruments are in each set?
   How many are there in all?

3. How else might you classify the instruments you found?

4. How many are in each set?

5. Find as many other ways of forming sets of instruments as you can.

   How many are in each?

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TEACHING NOTES: MATH LAB ACTIVITY CARDS

A-3 Sums (I-LI) Intermediate to
Late Intermediate
Level

Preparation

Make card available to learners working in pairs or in small groups, but have them do part or all of the exercises independently; then have them discuss their methods of solution.

NOTES:

Computation in the card will be either very tedious or relatively simple, depending on whether or not the learner makes use of the patterns in the tables. The teacher may be tempted to tell the learners how to avoid the tedious task, but by doing so, will deny the learner the opportunity to learn from his own experience. The learner who observes that R-1 + R-2 can be found in the first table by taking 5 x 11 and in the second table by taking 10 x 21 is well on his way toward appreciating the role of patterns in mathematics! Such appreciation is acquired most readily by learners who personally struggle with tasks that are made easy when he discovers the pattern!

These activities could be expanded to include the investigation of sums of sets of even numbers, sums of sets of odd numbers, or sums of sets of multiples of whole numbers such as:

10 + 20 + 30 + 40 +...........+ 100.

This last example provides excellent experience

MATH LAB ACTIVITY CARD

A-3

Sums

(I - LI)

R-1	1	2	3	4	5
R-2	10	9	8	7	6
R-3	11	12	13	14	15
R-4	20	19	18	17	16
R-5	21	22	23	24	25
R-6	30	29	28	27	26

1. If R-1 + R-2 means ADD all the numbers in row 1 and row 2, find these sums:

a. R-1 + R-2 ________
b. R-2 + R-3 ________
c. R-3 + R-4 ________
d. R-5 + R-6 ________
e. R-1 + R-2 + R-3 + R-4 ________
f. R-1 + R-2 + R-3 + R-4 + R-5 + R-6 = ________

2. What is the sum of:
a. The first 10 numbers? ________
b. The first 20 numbers? ________
c. The first 25 numbers? ________
d.. The first 30 numbers? ________

R-1	1	2	3	4	5	6	7	8	9	10
R-2	20	19	18	17	16	15	14	13	12	11
R-3	21	22	23	24	25	26	27	28	29	30
R-4	40	39	38	37	36	35	34	33	32	31
R-5	41	42	43	44	45	46	47	48	49	50
R-6	60	59	58	57	56	55	54	53	52	51
R-7	61	62	63	64	65	66	67	68	69	70
R-8	80	79	78	77	76	75	74	73	72	71
R-9	81	82	83	84	85	86	87	88	89	90
R-10	100	99	98	97	96	95	94	93	92	91

3. Find the sum of:
a. R-1 + R-2 ________
b. R-3 + R-4 ________
c. R-5 + R-6 ________
d. R-7 + R-8 ________
e. R-9 + R-10 ________
f. The first 100 numbers

TEACHING NOTES: MATH LAB ACTIVITY CARDS

A-3 Sums (I-LI) Intermediate to Late Intermediate Level

Notes (continued).......

with the distributive property--that is:

10 + 20 + 30 + 40 =
10 (1 + 2 + 3 + 4), and so forth.

ANSWERS:

1. a. 55
   b. 105
   c. 155
   d. 255
   e. 210
   f. 465

2. a. 55
   b. 210
   c. 325
   d. 465

3. a. 210
   b. 610
   c. 1010
   d. 1410
   e. 1810
   f. 5050

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TEACHING NOTES: MATH LAB ACTIVITY CARD

AS-1 Addition and Subtraction (EP) Early Primary Level

PREPARATION:

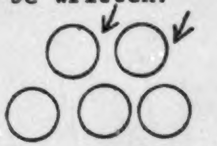
Provide learners with partitioned containers (such as egg cartons or some similar partitioned container) plus seven (7) marbles.

NOTES:

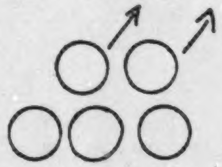
Allow learners to manipulate many kinds, sizes, shapes and colors of objects as well as themselves to represent joining and separating actions. As they learn, be certain that the children understand the operations of addition and subtraction of numbers exist independently of the materials utilized.

To clarify this, provide a variety of materials and experiences in order that learners may conclude that operations on numbers are merely represented by operation with sets.

Once children begin to see addition and subtraction as joining and separating actions, and when they can correctly manipulate materials and describe represented actions orally, it is time to describe the action in sentence form. For example, the actions described below could be written:



"Three dots with two dots equals five dots."

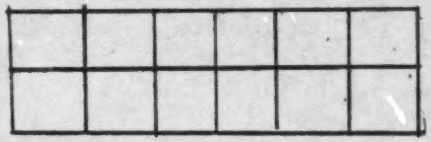


"Five dots less two dots equals three dots."

MATH LAB ACTIVITY CARD AS-1

Addition and Subtraction (EP)

1. Using the box with dividers in it, how many ways can you show five (5)? (Use only 5 marbles.)



Write an Addition sentence for each way:

2. How many ways can you show six (6)?

Write an Addition sentence for each way:

3. How many ways can you show seven (7)?

Write an Addition sentence for each way:

4. How many Subtraction sentences can you write for exercise C?

TEACHING NOTES: MATH LAB ACTIVITY CARD

AS-1 Addition and Subtraction (EP) Early Primary Level

In the early grades children are in the process of learning to read, and such written sentences tend to reinforce the need for reading. Also, written sentences are more descriptive of the action involved than are sentences using mathematical notation (such as, 3 + 2 = 5). Soon the teacher will want to introduce children to the more common mathematical sentences.

An early introduction to number sentences might involve duplication of a familiar situation similar to that described above. After writing the sentence in the familiar way, learners can be shown an easier way as to what has happened. Instead of writing "3 dots with 2 dots equals 5 dots," the learners can be shown that 3 + 2 = 5 describes the action. Emphasis should be made that this sentence (3 + 2 = 5) only describes the action; it does not tell the particular situation involved. That must be determined from the context of the problem. Additional situations should be available to learners which illustrate that the sentence, 3 + 2 = 5 correctly describes the action. Do the same with subtractive situations, showing that the number sentence only tells the action, not the specific instance.

Activity Card AS-1 is only an example of such situations.

After the idea of number sentences has been established, let learners make up a story to describe the action indicated. For example, an Activity Card could show the number sentence 3 + 2 = 5 and ask the learners to make up stories to show this type of activity. These stories could be told or written or otherwise illustrated. This serves as reinforcement as well as acquainting them with diverse problem settings calling for the same number sentence.

TEACHING NOTES: MATH LAB ACTIVITY CARD

AS-1 Addition and Subtraction (EP) Early Primary Level

Learners who can successfully perform activities of this type demonstrate a real knowledge of addition and subtraction as joining and separating operations. They are not likely to be confused with story problems they may encounter later. They become more discriminating of real situations that involve additive and subtractive situations within the daily conduct of the classroom and their day-to-day experiences outside of school.

TEACHING NOTES: MATH LAB ACTIVITY CARD

AS-2 Rationalizing Integers (I) Intermediate Level

PREPARATION:

Paper, a ruler, and crayon or pencils.

NOTES:

Intermediate learners have "informally" experienced many exposures to positive and negative integers.

Positive:
They have manipulated materials
They have written number sentences
They have solved everyday problems
Etc.

Negative:
They understand what is meant when one owes 5c
They know what is meant by "going into the hole" while playing a game
They know what is meant by 5° below zero

These experiences form a basis for more direct work in adding and subtracting integers.

Use of "number lines" with ACTIVITY CARDS can include the objectives learners need. This will necessitate developing a series of ACTIVITY CARDS which are aimed at positive and negative number experiences. CARDS can include problems to be solved using the number line:

> "In a bean toss game a miss counted as a "lost point." One player had the following points: (3, 1, miss, miss, 4, miss). Show on the number line what her score would be."

Rationalizing Integers (I)

1. How many days are there until Easter?

   Can you make a number line to show this?

2. Can you make a number line to show how many minutes it is until (lunch)?

   What happens when you get to 0?

3. Extend the number line you just made to show minutes after lunch.

   Show 5 minutes after lunch.

   Show 8 minutes.

   Make your line long enough to show 15 minutes after lunch.

4. How can you tell by looking at the number line whether a time is before or after lunch?

5. Where would you place the mark to show that I have 8¢ in my pocket?

   Where would you place the mark to show that I owe 5¢ (and other questions of similar kind.)

6. Show what would happen if I owe 5¢ and someone gives me 8¢.

   Make up another problem and show it on the number line.

7. Make a thermometer and show 6° below zero.

   Show what happens when the thermometer warms up 15° during the day.

TEACHING NOTES: MATH LAB ACTIVITY CARD

AS-2 Rationalizing Integers (I) Intermediate Level

NOTES: (Cont'd)

(Note: The use of the number line with "directed" or "signed" numbers, uses 0 as a "reference point" and the "direction of moves" is indicated by the sign of the numbers--negative means move to the left, positive means move to the right. Also, learners should avoid attempting computation with algorisms and statements of rules for manipulating positive and negative integers. This is a trap that many teachers allow and promote. Provide the learners with many active experiences with what integers do before emphasizing the algorism. In fact, learners will start developing such algorism if experiences are extensive.

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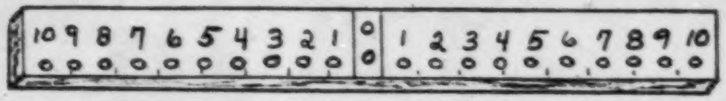
TEACHING NOTES: MATH LAB ACTIVITY CARDS

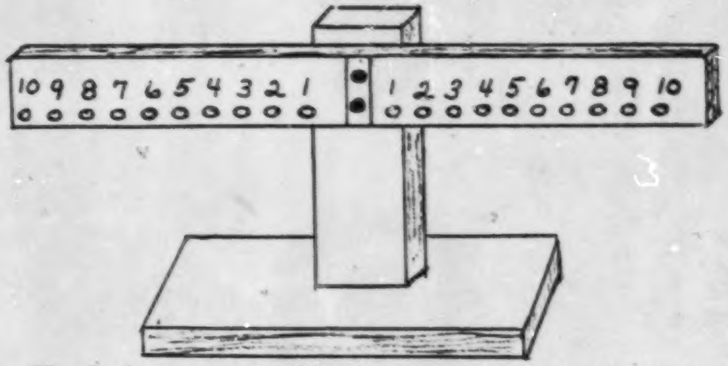
AP-1 Commutative Property (EP) Early Primary Level

Preparation:

Have available a simple set of weights and balances. These can be made as follows:



(Peg board, 21 or more holes long--have an odd number of holes because the middle holes will be used to support the balance.)



The holes may be filled with short dowels, and uniform objects such as metal washers can be hung on the dowels to represent the equations OR small hooks can be attached to uniform objects and hung in the holes to represent the equations.

NOTE: Commercially produced math balances are available and are more accurate than many homemade scales. Check School Supply catalogues and Publishers catalogues.

MATH LAB ACTIVITY CARD AP-1

Commutative Property (EP)

Use a set of weights and balances to help you match each addition sentence under A with the addition sentence under B which correctly completes the equation:

A	=	B
3 + 5		9 + 4
7 + 1		5 + 3
6 + 2		5 + 6
4 + 9		1 + 7
6 + 5		2 + 6

1. What did you discover?

2. Are there other addition equations which also balance?

3. Describe how your discovery helps you in mastering the basic addition facts.

AP-1 Commutative Property--Notes (continued).....

NOTES:

Using Addition Properties to help learn addition facts.

All of the 100 addition facts do not have to be memorized through repetitious drill activities. Many facts can be learned through the application of some properties of addition.

Commutative Property of Addition: "The order of the addends does not affect the sum."

This property is represented by the sentence:

$$a + b = b + a$$

For example:

$$4 + 5 = 5 + 4$$

If learners understand this property they are able to reduce the 100 basic addition facts by almost one-half.

To help learners discover the commutative property of addition, Activity Cards like AP-1 can be used.

Identity Element of Addition: "The zero property of addition--adding 0 to any number leaves the quantity unchanged."

Applying this principle allows learners to reduce the number facts to memorize by 10.

Application of both of the Properties--Commutative and Identity--to Subtraction.

Learners should be led to explore whether these properties can be applied to subtraction or not. If explored, they will discover that the operation of subtraction is not commutative:

$$9 - 4 \neq 4 - 9$$

Thus, learners will not be able to reduce the independent subtraction facts to be learned materially.

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TEACHING NOTES: MATH LAB ACTIVITY CARD

AP-2 (MP) Middle Primary

Computation Patterns (Associative and Commutative Properties for Addition

Preparation

Provide card for learners to work independent of the teacher. Have learners work in pairs or small groups.

NOTES

When tables are complete, encourage sharing of their insights and different ways they found the sums. The teacher will avoid the temptation to introduce the formal language of math if the teacher assumes the role of "drawing out the learners' insights."

Learners might be urged to construct similar tables for each other, or others that focus on larger numbers.

(Answers)

A = 18	D = 21
B = 27	E = 45
C = 27	F = 30

MATH LAB ACTIVITY CARD

AP-2
(MP)

Computation Patterns (Associative and Commutative Properties of Addition)

Find the sum of the numbers in each table:

A.

1	2	3	
2	3	1	
3	1	2	
			Sum

B.

1	3	5	
3	5	1	
5	1	3	
			Sum

C.

2	3	4	
3	4	2	
4	2	3	
			Sum

D.

1	2	4	
2	4	1	
4	1	2	
			Sum

E.

4	5	6	
5	6	4	
6	4	5	
			Sum

2	3	5	
7	2	1	
1	5	4	
			Sum

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TEACHING NOTES: MATH LAB ACTIVITY CARD

M-1 Introduction to Multiplication (P) Primary Level

PREPARATION:

Make available boxes of blocks.

NOTES:

The need for multiplication will be discovered by learners as they develop a background in addition whereby they have been encouraged to manipulate objects and to search for patterns.

Such an ACTIVITY CARD as M-1 provides for the manipulation of physical objects which will lead to many patterns of multiplication. The teacher's role in such explorations is to provide the opportunity for learners to discover solutions! (NO explanations; no highly structured examples of what multiplication means.) The teacher (if present) should accept learners' answers, but alternatives they might examine could be suggested:

> "How many different ways can you get sets of equal size from any number of blocks? Make a chart of your findings."

Providing more information than is needed usually leads to learner verbalization. For instance, if the teacher pointed out that 2 x 2 is just 2 more than 1 x 2 and so on, could seemingly be understood by learners, but the explanation of such a principle has not been accompanied by the learner's manipulation and experiencing in order for him to discover such a principle, thus he really has not gained that concept and will not be able to use it in subsequent work.

MATH LAB ACTIVITY CARD M-1

Introduction to Multiplication (P)

Using the box of blocks provided do the following:

1. Using your blocks, find as many addition combinations as you can to show 12.
   Stop if you find more than 15.
   Write an addition sentence for each.

2. Find as many combinations as you can using sets of equal size, such as

   ☐☐☐☐ ☐☐☐☐ ☐☐☐☐
   and so on, which show 12.

   Draw a sketch of your answers.

   Write an addition sentence for each.

3. Using 13 blocks, find as many combinations as you can using sets of equal size.

   Write a number sentence for each.

   Do the same with 14 and 15.

   Do the same with sets of 10 and 11.

   What have you noticed?

4. Using sets of blocks from 1 through 9, find as many combinations as you can using sets of equal size.

5. Make a chart to record your findings.

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TEACHING NOTES: MATH LAB ACTIVITY CARD

M-1 Introduction to Multiplication (P) Primary Level

NOTES: (Cont'd)

Left to their own initiative, learners will quickly discover patterns for themselves. They can make discoveries of many of the following properties if provided with ACTIVITY CARDS and exploration time: (The teacher will often aid their identifying a name for the property so they have a common vocabulary for properties they discover).

Commutative property
identity element
zero property

Many other activities which can be used to introduce multiplication can be put into ACTIVITY CARDS.

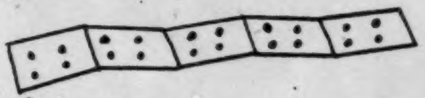
Attribute activities (wherein the sets of equal size have been pre-structured).

Multiple counting experiences especially using objects, number lines, flannel board pieces, etc.

Playing cards

4 of each: aces, tens, etc. within the set of aces there are 2 subsets of black and red.
13 hearts, etc.
Many questions could be posed to lead learners to discover "sets of equal sizes" in a deck of playing cards.

Folded oaktag "dot pages" can stimulate interesting discoveries:



etc.

Geo-Boards
Put rubber bands around all sets of equal size (2,3,4,etc)

Array Boards
Place rubber bands to show sets of equal size on different Boards.

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TEACHING NOTES: MATH LAB ACTIVITY CARDS

M-9 Multiplication

Preparation

5 X 5 or larger array Geo-Board
Colored Rubber bands.

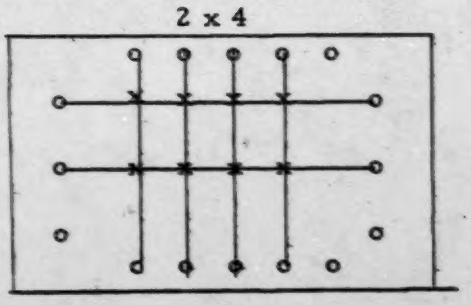
NOTES:

Card M-9 is related to the Function Cards F-1 through F-6, since a multiplication array like 2 x 4 is a Function.

2 (number of sets)
4 (size of sets)

Total number in the sets is a Function of the number and size of sets.

A special kind of Geo-Board called the Geo-Array-Board is best for showing multiplication arrays. It looks like this:



The x intersections indicate the multiplication.

MATH LAB ACTIVITY CARD M-9

Multiplication

1. How would you show 2 x 4 on the Geo-Board?

2. How would you show 3 x 5 on the Geo-Board?

3. How would you show 1 x 3 on the Geo-Board?

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TEACHING NOTES: MATH LAB ACTIVITY CARDS

MP-4 Distributive Property--Multiplication (I) Intermediate Level

Preparation:

1. Make card available to learners.
2. Have learners find the sum for each table.
3. Have learners work in pairs or small groups. When the sum is found, draw out as many different techniques as possible for finding the sum. See if someone comes up with a "shortcut" in Table A using the Distributive concept--sum 110 could be found from the bottom row:

$$(11 \times 3) + (11 \times 5) + (11 \times 2) = 11 \times 10$$

OR the right column:

$$(2 \times 10) + (4 \times 10) + (5 \times 10) = 11 \times 10$$

Have learners discuss the patterns in each other's tables.

NOTES:

The distributive property is the most elusive of the basic properties for most learners. This card will give learners an opportunity to identify the distributive pattern and see how it provides for efficient computation.

Some learners will have had only a pattern-oriented experience involving application of the multiplication basic facts and some addition.

ANSWERS:

a. 110
b. 210
c. 100
d. 120

MATH LAB ACTIVITY CARD

MP-4

Distributive Property--Multiplication (I)

Find the sum of the numbers in each table:

A.

2 x 3	2 x 5	2 x 2
4 x 3	4 x 5	4 x 2
5 x 3	5 x 5	5 x 2

Sum

B.

3 x 4	3 x 5	3 x 6
5 x 4	5 x 5	5 x 6
6 x 4	6 x 5	6 x 6

Sum

C.

2 x 1	2 x 3	2 x 6
3 x 1	3 x 3	3 x 6
5 x 1	5 x 3	5 x 6

Sum

D.

1 x 5	1 x 7	1 x 8
2 x 5	2 x 7	2 x 8
3 x 5	3 x 7	3 x 8

Sum

CMM:006:05 (ISU) Teaching Notes: Math Lab Activity Card
GR-2 "Addresses" (MP)
Middle
Primary
Level

Background:

No preparation for learners needed beyond making the card available.

NOTES:

Since this card involves work with negative numbers and with coordinates, the top of the card shows:

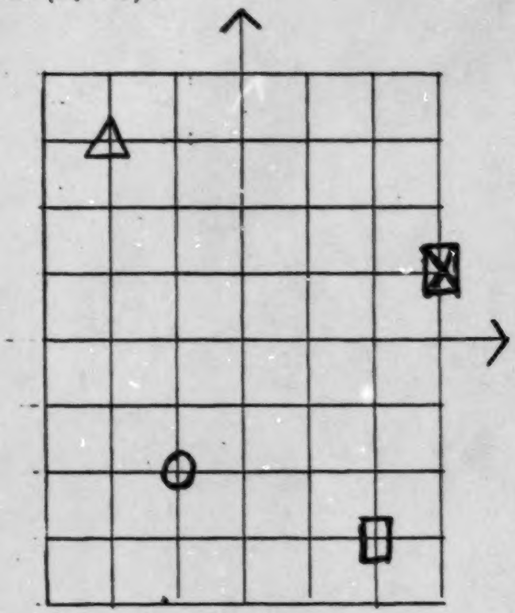
The Square Shape is at (2, 3).
The Address of the box with X is (3, 2).
The Origin where the major lines intersect is at (0, 0).

The bottom of the card suggests using the game of tic-tac-toe on a grid like the one illustrated to provide practice in plotting these ordered pairs of numbers. You may want to organize this as a class game, dividing the whole class into two teams.

By convention, the 1st number given indicates the east-west distance from the origin. It is best that the learners use this convention from the start.

MATH LAB ACTIVITY CARD
"Addresses"
GR-2
(MP)

The △ is at (-2, 3).
The ○ is at (-1, -2).
What Shape is at (2, -3)?



Can you figure out the address of the X ?
Put a ● at (-3, -4); a ▲ at (-3, 4); and a ■ at (0, 0).

Make another graph like the one above. Play tic-tac-toe with a partner. Follow these rules:
1. You need 4 in a row to win.
2. Suppose you put an X and your partner put an 0. Then you give an address to your partner and he puts your X on the grid.
3. You put your partner's 0's on the address he gives you.

Addresses like these are called Coordinates.

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TEACHING NOTES: MATH LAB ACTIVITY CARDS

F-1 F-4
F-2 F-5 Functions
F-3 F-6

Preparation

5 X 5 Geo-Board
Dot sheets
Graph paper
colored rubber bands

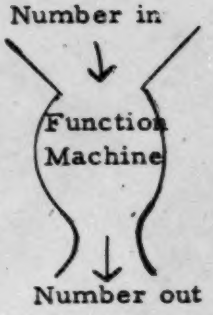
NOTES:

Card F-1 reveals a pattern of even numbers.

Card F-2 introduces the learner to Functions. This card can be introduced by use of the game "Guess My Rule" or "What's My Rule?" or "Guessing Functions." It goes like this:

"I have a machine at home. I put a number into it, press the button and out comes another number. You have to figure out how my rule works."

"The number you tell me to put in is classified (on Card F-2) as the ☐ number, and the number that comes out of the machine is the △ number. Give a number to put into the machine. (Learner may respond, "Three.") If we put 3 into the machine it gives out five.



MATH LAB ACTIVITY CARD F-1

Functions

What do you notice about the number of dots?

1. o
o

2. o o
o o

3. o o o
o o o

4. o o o o
o o o o

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TEACHING NOTES: MATH LAB ACTIVITY CARDS

F-1 F-4
F-2 F-5
F-3 F-6

Functions

	□	△	
	3	5	
Put in	7	9	Machine
machine.	5	7	gives out.

If we put 7 in, 9 comes out. Give me another number to put in. (Learner may respond, "5.")

What will the machine give out? (Learner responds "7.") Seven is right.

If you think you know my rule, do not tell, but give me a number to put in the machine and give the number which comes out!

(The teacher should solicite the learners to repeatedly apply the rule they think they know with in-out numbers. The teacher should avoid urging learners to come to closure; that is, tell the rule. This allows many learners to experience the patterns (or Functions) repeatedly before the attempt to generalize. This kind of learning experience will aid them in testing and trusting their own thinking!)

After a wh8le the teacher could encourage the learners to write the rule. (Some will do this in words but it eventually should be written using mathematical symbols, such as:

$$\square + 2 = \triangle$$

MATH LAB ACTIVITY CARD F-2

Functions

Below is a table to keep track of the number of dots in each case (from Card F-1):

Number of Dot Pairs □	△ Number of Total Dots
1	2
2	4
3	
4	
.	
.	
.	
10	

1. Can you complete the table?

2. Can you write a rule or open sentence relating the □-number and the △-number?

TEACHING NOTES: MATH LAB ACTIVITY CARDS

F-1 F-4
F-2 F-5
F-3 F-6

Functions

Then it should be tested to see if it works for all the pairs of numbers in the table--Card F-2

Now, learners can do additional problems (making some Math Lab Activity Cards is a good way) by making up a rule (they should write an open sentence) and try it on others. This will lead to "graphing functions" which is included in Cards F-3 and F-6.

Suggestions of sources for other functions problems:

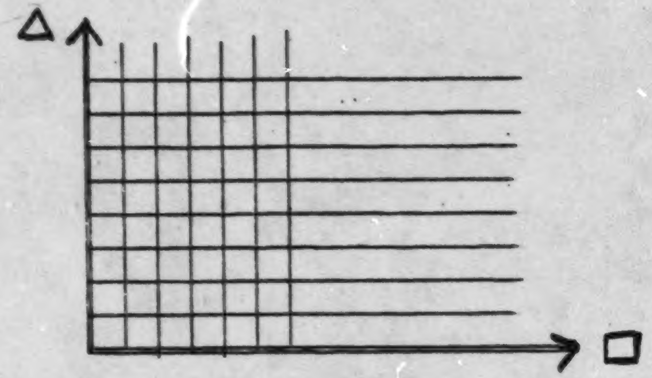
Natural physical settings--centigrade and
farenheit temperatures
Weights on a spring and sand pendulum.
From rods
From blocks
From Geo-boards
(Cards F-1, F-2, F-3, F-4, F-5, F-6)

Card F-3 introduces graphing of Functions.
Card F-4 reveals a pattern of ODD numbers.
Card F-5 deals more with functions.
Card F-6 deals more with graphing functions.

MATH LAB ACTIVITY CARD F-3

Functions

Can you graph the pairs of numbers in the table on Card F-3 on a piece of graph paper?



F-1 F-4
F-2 F-5
F-3 F-6

Functions

ANSWERS FOR TABLES AND GRAPHS

Card F-2

Dot Pairs

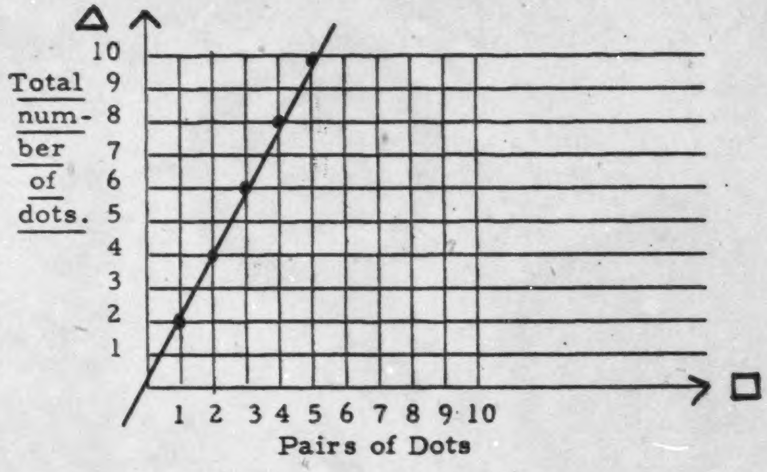
□	1	2	3	4;	....	10
△	2	4	6	8	....	20

Total Dots

Rule: Add 2 to □ to get △, or add a pair of dots to □ to get △.

Open Sentence: △ = □ + 2, OR
△ = □ + pair of dots.

Card F-3



NOTE: Graph of the even numbers forms a straight line-- as □ increases by 1, △ increases by 2.

MATH LAB ACTIVITY CARD F-4

Functions

What do you notice about the number of dots below?

1. o o
o

2. o o o
o o

3. o o o o
o o o

4. o o o o o
o o o o

TEACHING NOTES: MATH LAB ACTIVITY CARDS

F-1 F-4
F-2 F-5 Functions
F-3 F-6

Card F-4:

Increase by a pair each time.

Card F-5:

Pairs of Dots

□	1	2	3	4	....	100
△ Total Dots	3	5	7	9	....	201

NOTE: Pairs of Dots really start with three ($\begin{smallmatrix}o&o\\o&\end{smallmatrix}$), thus 2 pairs really means $\begin{matrix}o&o\\o&\end{matrix} + \begin{matrix}o\\o\end{matrix}$ (2 more)
since the pattern increases by 2 each time.

MATH LAB ACTIVITY CARD

F-5

Functions

Let □ be the number of pairs of dots and △ be the number of total dots. (Refer to Card F-4.)

1. Can you complete this table?

2. Can you write a rule relating the □ - numbers and the △ - numbers?

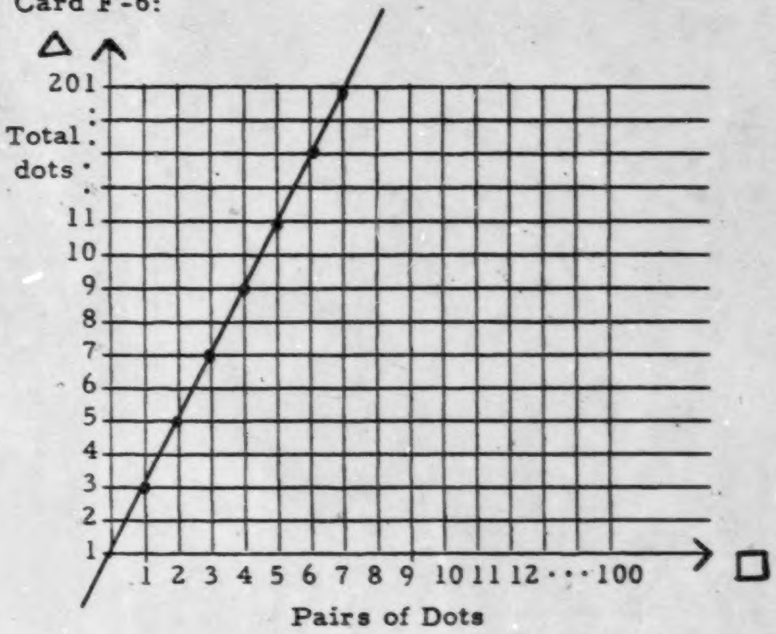
□	△
1	3
2	5
3	.
4	.
.	.
.	.
.	.
100	

TEACHING NOTES: MATH LAB ACTIVITY CARDS

Functions

F-1
F-2
F-3
F-4
F-5
F-6

Card F-6:

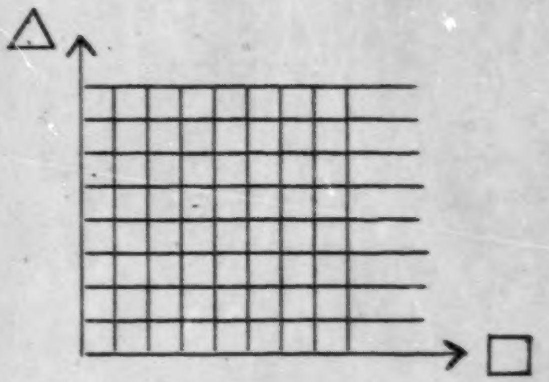


NOTE: Graph of the Odd numbers forms a straight line---as □ increases by 1, △ increases by 2.

MATH LAB ACTIVITY CARD F-6

Functions

1. Can you graph the pairs of numbers in the table (see Card F-5) on a piece of graph paper?



2. What kind of graph do you get?

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TEACHING NOTES: MATH LAB ACTIVITY CARDS

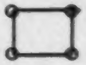
G-12 How Many Squares?

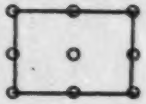
Preparation

5 x 5 Geo-Board
Colored rubber bands

NOTES:

This card seems to be asking a single question that would not require much activity, but squares will have different numbers of pegs, such as:



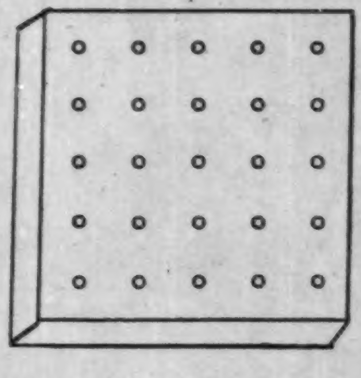


(See if you can find the 30 squares. If you find more, collect a prize for "genius.")

MATH LAB ACTIVITY CARD G-12

How Many Squares?

How many squares can you construct on the Geo-Board?



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TEACHING NOTES: MATH LAB ACTIVITY CARDS

G-13 through
G-21

Polygons

Preparation

5 x 5 Geo-Board
Colored rubber bands
Dot paper

NOTES:

Work with Polygons help learners experience many patterns when these experiences employ standardizing devices like the Geo-Board or Dot paper. These devices provide a measured distance between Dots of Pegs which allows for more accurate comparisons, thus discovering patterns becomes easier.

This series of cards (G-13 through G-21) also develops a readiness for naming similar shaped figures.

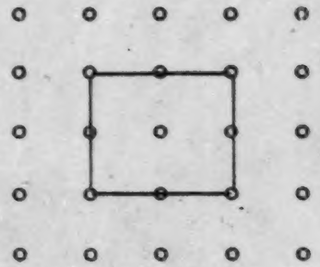
Terminology is only important when learners begin attempts to standardize their language when referring to repeated examples of shapes or processes. This need to be precise in calling certain figures squares because they have particular attributes (4 equal sides, 4 right angles...), is at the heart of one of the main goals of modern math--"preciseness of language."

There are also a number of applications of polygons involved in these cards--shapes in construction and shapes in nature.

MATH LAB ACTIVITY CARD G-13

Polygons

1. Make this square on the Geo-Board.



2. Can you make a figure which has 3 sides?

3. Can you make a figure which has more than 4 sides?

Keep track of your figures on a piece of Dot paper.

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TEACHING NOTES: MATH LAB ACTIVITY CARDS

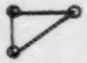
G-13 through
G-21

Polygons

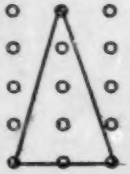
The teacher is urged not to give names of the shapes until learners are able to identify their attributes and are unable to find the names themselves. This requires a great deal of suspended judgement. (Just bite your tongue and say to yourself, "It's the learner who is trying to do the learning, not me.") If you try to short circuit the learner's attempts to learn something you are increasing his dependence on you to validate his own thinking rather than trusting and testing his own thinking.

Card G-15

It would be very tempting on the teacher's part to supply the learner the special names for certain triangles made while exploring Card G-15. DON'T DO IT. Instead, encourage the learner to look at the attributes of various three sided figures:



2 sides same length
1 side longer
(isosceles)



Also isosceles, but
notice the changing angles.

MATH LAB ACTIVITY CARD

G-14

Polygons

1. How many different shape figures with four (4) sides can you make on the Geo-Board?

   Keep track of these shapes on Dot paper.

2. Do any of these shapes have special names?

3. Can you find any of these figures in your classroom, at home, or outside?

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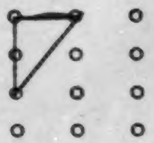
TEACHING NOTES: MATH LAB ACTIVITY CARDS

G-13 through
G-21

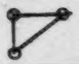
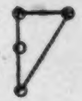
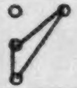
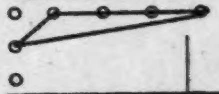
Polygons



Now all sides are different lengths. (Can this be true without the one right angle?)

There are many excellent probings which the teacher can encourage without becoming an "answer seeker."

If a learner suggests that he has found all the possible different 3-sided figures, you might encourage him to make a table showing all the attributes of the different 3-sided figures.

Different 3-sided figures	Attributes 2 angles alike; 1 different	Has 90° angle	All sides different	1 angle more than 90°	Etc.
	X	X			
		X	X		
	X			X	
			X	X	

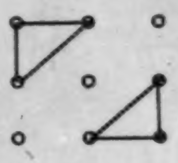
MATH LAB ACTIVITY CARD G-15

Polygons

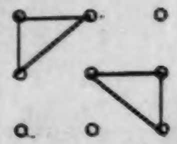
1. How many different three (3) sided figures can you construct on the Geo-Board?

   Keep track of these on Dot paper.

NOTE: Different means that NO three-sided figure will be alike.



These are alike!
(Mirror images)



These are alike!
(rotated images)

2. Do any of these figures have special names?

3. Can you find any of these figures in your classroom, at home, or outside?

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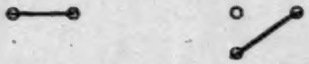
TEACHING NOTES: MATH LAB ACTIVITY CARDS

G-13 through
G-21

Polygons

Card G-16:

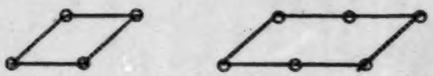
The reason card G-16 suggests that the Pentagon Building is almost the shape shown on the card is that a Geo-Board does not have the same lengths between pegs when you go from dot-to-dot in these two ways:



The "Pentagon" on Card G-16 is irregular. All other 5-sided shapes that learners attempt will also be irregular.

Card G-17:

The baseball plate is a regular pentagon--all 5 sides the same length. The diamond is merely a "rotated square"--its angles are all 90° and its sides are all equal in length. A "parallelogram" is a pushed over square; that is, its opposite angles are equal but none are 90°. It also may be a pushed over rectangle:

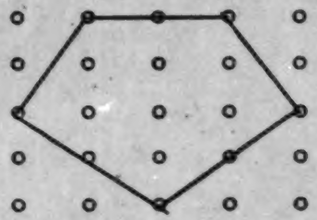


Learners can discover these and many other attributes. Teachers should encourage "looking for patterns" in work with polygons on the Geo-Board and this often requires the learner to organize his data on Dot sheets.

MATH LAB ACTIVITY CARD G-16

Polygons

There is a famous building in the United States which has 5 sides. It "almost" looks like this:



1. Can you find out its name and tell where it is?

2. Can you make any other 5-sided figures on the Geo-Board?

3. Are any of them different from the one above?

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CMM:006:05(ISU) TEACHING NOTES: MATH LAB ACTIVITY CARDS

G-18: Polygons

This card is a kind of riddle or puzzle. No hints should be given, except possibly "Go out and look around." The STOP SIGN is a standardized shape (as are many directional signs used by traffic systems throughout the United States and the World.

The BEE HIVE or CELL is a phenomena in nature. There are many such phenomena--crystal shapes, snow flakes, spider webs, arrangements of leaves and flower petals of plants and trees, etc. These "shapes" should trigger many explorations with the naked eye and the magnifying glass or microscope. Cell structures have many regular and irregular shapes.

G-19, G-20, and G-21: Polygons

This CARD is more of a "stimulus to Research" because the learner is being asked to search for information which describes the uses of the triangular shape.

(CAUTION: The teacher must avoid responding to learners with answers to questions included on the cards. Setting the conditions with learners to support their inquiry into these cards is most important if learners are to develop and use skills of searching, collecting, and reporting data. Also, these cards provide opportunities for learners to apply skills of locating and experimenting with the contents of the cards.)

Teachers may find that other cards could be written which are more appropriate for a particular individual or group. Learners should be encouraged to "generate" some LAB CARDS themselves in order to try out some new ideas or discoveries they have made. In time, a file of LAB CARDS should grow to rather refined levels, but it would be important to "weed" as you add to such a file.

MATH LAB ACTIVITY CARD G-17

Polygons

1. There is a famous object which has 5 sides, is made of rubber, is hit, run over, swept, stepped up to, and is a kind of dish. Can you guess what it is?

2. There is a very famous square. It is not called a square and is 90 feet on each side. Can you guess what it is?

3. You might try to make a drawing of either or both of the above objects by using what you know about the shapes of these objects.

Page 6 of 7

MATH LAB ACTIVITIY CARD

G-19

Polygons

1. Why are TRIANGLES used in the construction of many bridges?
2. See if you can find out how a TRIANGLE can be used to find the height of a building.
   Report your findings.

MATH LAB ACTIVITY CARD

G-18

Polygons

1. There is a well-known 8-sided polygon which usually stands around street corners. Do you know what it is?
2. What is a sweet hexagon? Do you know where to find them?

Page 7 of 7

MATH LAB ACTIVITY CARD G-20

Polygons

1. What is a T-SQUARE and how is it used?

2. How is CARPETING sold?

3. How is OFFICE SPACE rented?

4. Find out what you can about the "GOLDEN RECTANGLE." Report your findings.

MATH LAB ACTIVITY CARD G-21

Polygons

1. What are QUADRUPLETS?

   What is a "LATERAL" in football?

   What is a QUADRILATERAL?

2. Find out the meanings of 4 other words which have the Prefix "Tri" as in Triangle:

3. Can you make up any other problems?

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TEACHING NOTES: MATH LAB ACTIVITY CARDS

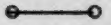
LS-1 Line Segments---Congruence

Preparation

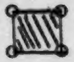
5 x 5 Geo-Board
Colored rubber bands

NOTES:

When dealing with line segments on a Geo-Board, the unit of length will be the "non-diagonal distance between two adjacent pegs."



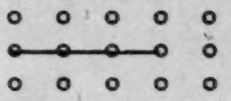
The unit of area will be this region:



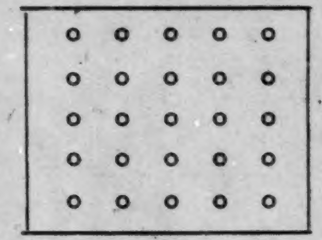
Any line segment which can be moved so that all of its points coincide with another is said to be congruent to the other one.

MATH LAB ACTIVITY CARD LS-1

Line Segments---Congruence



How many line segments, the same length as the one shown above, can you find on the Geo-Board?



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TEACHING NOTES: MATH LAB ACTIVITY CARDS

LS-2 Line Segment---Congruence

Preparation

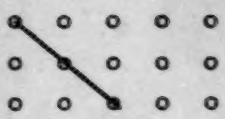
5 x 5 Geo-Board
Colored rubber bands
Dot paper and pencil

NOTES

This card will require considerable exploring and learners will not all come up with all the responses possible. This should not be a cause for concern. Different answers allows different learners to be successful though all possibilities are not found.

MATH LAB ACTIVITY CARD LS-2

Line Segment---Congruence



1. How many line segments the same length as the one shown above can you find on the Geo-Board?

2. How many line segments of different lengths can you find on the Geo-Board?

3. How many of each length are there on the Geo-Board?

CMM:006:05 (ISU) Teaching Notes: Math Lab Activity Card

PA-1 "What's My Rule?" (M)
(Patterns) Middle Grade Level

Background:

This card deals with a "division" rule but no preparation or directions are needed except making the card available.

NOTES: Top of the card:

Rule of Division, fully stated is "Divide the number of the column by the numer of the row." Notice how this rule agrees with the convention dealing with how to read ordered pairs (coordinates)--1st number given indicates east-west direction from the origin.

Bottom of the card:

Asks if the learner can make up a rule that is too hard for anyone in class, including the teacher, to solve. Do not feel threatened by this. The essence of scientific thinking is the attempt by very brilliant thinkers to "figure out the rule," and these men and women fail much more often than they succeed. Don't always expect to be able to figure out the rule, and don't limit your thinking to questions whose answers you are sure of!

MATH LAB ACTIVITY CARD
"What's My Rule?"

PA-1
(M)

Can you figure out the rule for this chart?

	12	24	36	48	60	1	
1							
2			18				
3	4				20		
4		6				1/4	
6							
12							

Can you fill-in the rest of the chart?

Can you make up a rule that is too hard for anyone in the class, including the teacher?

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TEACHING NOTES: MATH LAB ACTIVITY CARDS

PA-2 Numeration Patterns (EP) Early Primary Level

Preparation

Make card available to learner without much teacher direction. Start learners off by stating:

> "Each table is to have numbers 1 to 16 in it. Some numbers are missing. See if you can decide what number to put in each blank space."

NOTES:

Variations on this experience might include using a 3 x 3 table for beginners or starting the counting sequence with some number other than 1 for more advanced learners.

MATH LAB ACTIVITY CARD PA-2

Numeration Patterns (EP)

Fill in the blanks in each table.

A.

1		3	4
5	6		8
9	10		12
13			16

B.

1	2		4
8		6	5
9			12
16			13

C.

1		9	
2	7	10	15
4	5	12	13

D.

13		15	
12	11		9
5	6		
	3	2	1

E.

		14	13
9			12
	7		5
	2	3	

F.

	9		1
15		7	
	11		3
13		5	4

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TEACHING NOTES: MATH LAB ACTIVITY CARDS

PA-3 Free Play---Geo-Board All Levels

Preparation

5 X 5 array Geo-Board
Colored rubber bands
Dot paper and pencil

NOTES:

Allow free play without teacher direction at first; observe to gain insight into ways learners "try ideas." Many clues can be gained about the styles of attacking problems which learners reveal.

Learners will not always see or verbalize what you see! Don't force it.

MATH LAB ACTIVITY CARD PA-3
(All)

Free Play---Geo-Board

Using a Geo-Board, colored rubber bands and Dot paper:

1. Make a design on the Geo-Board.
2. Copy your design onto a piece of Dot paper.
3. How is your design the same as or different from a design made by a classmate?

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TEACHING NOTES: MATH LAB ACTIVITY CARDS

PA-4 Copy Figures All Levels

Preparation

5 X 5 Geo-Board or Geo-Board with
10 X 10 array.
Dot sheets and pencil
Colored rubber bands

NOTES:

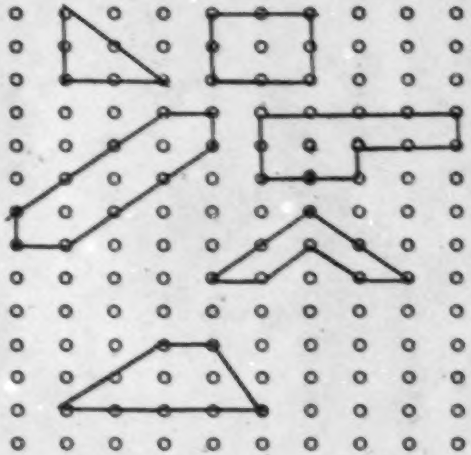
Some learners will discover that some figures have pegs inside the figures and some do not.

Learners may count how many pegs each figure contains.

MATH LAB ACTIVITY CARD PA-4

Copy Figures (All)

Can you make these figures on your Geo-Board? On Dot paper?



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TEACHING NOTES: MATH LAB ACTIVITY CARDS

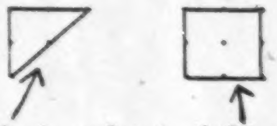
PA-5 Pegs Within Figures All Levels

Preparation

5 X 5 Geo-Board
colored rubber bands

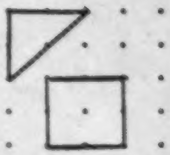
NOTES:

The patterns involved in comparing shapes begins to develop ideas of sets, subsets, more than, less than, etc.


This is subset of this.

MATH LAB ACTIVITY CARD PA-5

Pegs Within Figures (All)



1. How many pegs are inside the triangle?

2. How many pegs are inside the square?

3. How many pegs do the triangle and the square have in common?

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TEACHING NOTES: MATH LAB ACTIVITY CARDS

C-1 Counting---sets (P) Primary Level

Preparation

5 X 5 Geo-Board
colored rubber bands

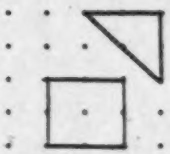
NOTES

Rather than just counting the pegs on the Geo-Board, various counting activities can be done which are related to some mathematical concepts.

This allows counting to find how many to become more meaningful.

MATH LAB ACTIVITY CARD C-1 (P)

Counting---sets



1. How many pegs are inside the triangle?

2. How many pegs are inside the square?

3. How many pegs do the triangle and the square have in common?

OVERLAP the triangle and the square to find out.

CMM:006:05 (ISU) Page 1 of 1

TEACHING NOTES: MATH LAB ACTIVITY CARDS

TO-1 TO-3
TO-2 TO-4 How Many Routes?

Preparation

5 x 5 Geo-Board
Colored rubber bands
2 colored markers-- ⊡ ⊙
Dot paper and pencil

NOTES:

Topology experiences are not only interesting but provide many challenges for any learner.

The TO-1 card is the first in a series of cards designed to build an understanding of Pascal's Triangle.

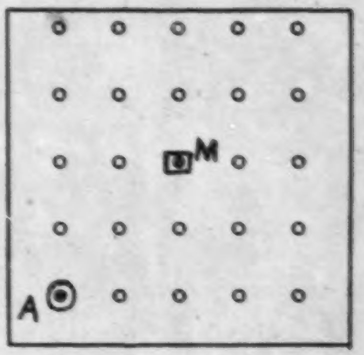
Pascal's Triangle is the pattern that results, writing the number of routes a little differently:

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
. 5 10 10 5 .
. . . .
. . . .

This same pattern comes up in probability. (If you flip 5 coins, how many ways can 3 heads and 2 tails come up?) The pattern also shows up when you square a binomial $(a + b)^2 = \underline{1}a^2 + 2\,ab + \underline{1}b^2$. (See: Fardner, "Mathematical Games" in Scientific American, December, 1966.)

MATH LAB ACTIVITY CARD TO-1

How Many Routes?



Al lives at point "A" and Marty lives at "M."

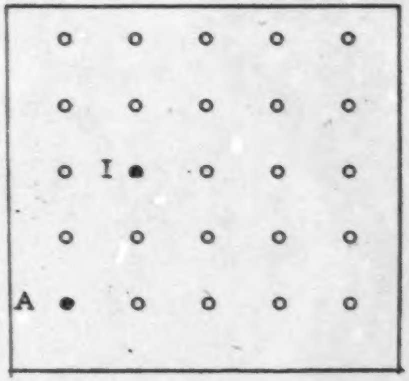
Al can only travel to the right or up. (He cannot travel to the left or down or diagonally.)

How many routes can Al take to go to Marty's house?

MATH LAB ACTIVITY CARD

TO-2

How Many Routes?



Ira said Al could take 2 routes to get to his house.

Al said he could take 3 routes.

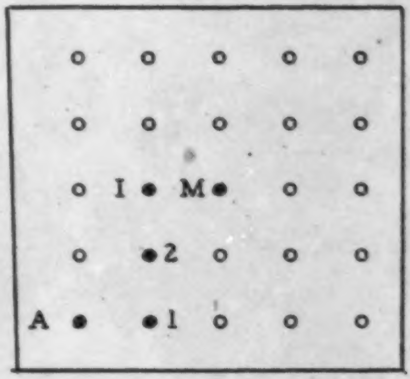
Who is right?

MATH LAB ACTIVITY CARD

TO-3

How Many Routes?

Use Dot Paper:



Place the number of routes Al can take to each house (Ira's and Marty's) on a piece of Dot paper with the numeral near each point.

What Do You Notice?

MATH LAB ACTIVITY CARD **TO-4**

How Many Routes?: Pascal's Triangle

This is a pattern of the number of routes you have worked with on Cards TO-1, TO-2 and TO-3.

How is it related to what you have done on the other cards?

```
              1
            1   1
          1   2   1
        1   3   3   1
      1   4   6   4   1
    1   5  10  10   5   1
  .   .   .   .   .   .   .
.   .   .   .   .   .   .   .
```

MATH LAB ACTIVITY
BALANCING-BLOCK MATH

NOTE: This is a Math-science activity.

HOW TO BEGIN: (Materials)	RATIONALE:	HOW TO DO IT: (Activities)
Comparing things--size, color, and shape--the effects of objects positioned on a board which is balanced on a fulcrum.	This unit intends to involve learners to countless experiences in which they control their involvement in an experimental situation.	Activities for this activity center around WRITTEN QUESTION SHEETS developed by the teacher OR verbal questions periodically stated by the teacher.
Series of confrontations in which blocks, a fulcrum and a board become first-hand experiences. BOARD: plywood ½" x 3" x 24" FULCRUM: dowel 1½" diameter, cut in half and 4" long	This unit has few facts, thus its purpose is not to teach facts but implies much more than just learning facts.	Learners should RECORD their findings. After a while children should share their various RECORDING METHODS. The class may even decide on a COMMON PROCEDURE FOR RECORDING their findings.
LIGHT BLOCKS: 1" x 1" x 1" Rule the BOARD in equal divisions with a magic marker--mark middle first and then mark every 2". (Mark edges every 2" also and along one edge number from each side of the middle 1, 2, 3, 4, 5, and 6. Do not number the middle line.) (NOTE: Optional or alternate BALANCING BOARD could be 40" long and use 10 blocks--this would more aptly relate to the base 10 number system.)	Science and math teaching must give learners a chance to make mistakes, to "cut and try" and to develop confidence which is not controlled primarily by teacher approval. It is important in this unit for the teacher to BE PATIENT; give children the opportunity to gain experience in the balancing situation, and by doing so accomplish in a long time what you may feel would be done in considerably less time.	Give a BOARD and FULCRUM TO each child or small group (2 or 3). State the following QUESTION: "Can you balance the board on the fulcrum?" (NOTE: Allow learners to place board on fulcrum flat, on edge, or on end WITHOUT TEACHER INTERFERENCE OR HELP. You asked a question which by its very nature IMPLIES INDIVIDUAL RESPONSES. These responses NEED NOT agree with teacher's preconceived notion of how to balance the board flat on the fulcrum. Once the blocks are introduced, the children will balance the board flat on the fulcrum).

HOW TO DO IT: (Activities)

Hand out the BLOCKS--8 or so to each learner--and let them "PLAY." (NOTE: For convenience in handling and passing out material, place in separate bags the blocks for each learner.)

Now starts the SERIES OF ENCOUNTERS in which children gain experience with the balancing board and blocks.

The questions gradually become more difficult.
Each question will require varying degrees of TIME and PRACTICE for each child.
Do NOT keep the learners together in "lock step."
WRITTEN LESSON SHEETS can be developed using the following questions. (NOTE: Such a LESSON SHEET is attached, but it is ONLY meant to be a sample, NOT "the" lesson sheet.)
(NOTE OF CAUTION: It is better to have each learner work for a short time (30-35 minutes) 2 or 3 times a week on these questions rather than attempt to rush through them in a few prolonged periods. Give time for reflection on things outside the classroom; given time, learners will observe applications of the board and fulcrum outside the class.)

QUESTIONS: (See the sample WRITTEN LESSON SHEET which lists the questions. Below are SUGGESTIONS FOR UTILIZING the questions to their fullest):

SUGGESTIONS FOR UTILIZING QUESTIONS:

A. Questions 1, 2, 3, 4, 5, 6 and 7: Provide specific directions as to the number of blocks necessary for each experience. Initially the learners will experiment with the same number of blocks on each side of the fulcrum (questions 2, 3 and 4) while later they will experiment with different numbers of blocks on either side of the fulcrum (questions 5, 6 and 7).

Encourage the learners to obtain many responses to each question. Their answers can be recorded on a WORKSHEET which has drawings of the board and fulcrum. (See attached sample WORKSHEET.)

NOTE: The learners will try to hurry through the questinns, for they have probably always equated number of questions answered with success--a good grade. SLOW the LEARNERS DOWN by getting them to try several different balancing combinations RECORDING EACH) for each question. The children are gaining experience with the

A. (continued).... "balancing situation" during the early questions (1-7). The TEACHER'S ROLE is to create an ATMOSPHERE THAT WILL PERMIT THIS INDEPENDENT INVESTIGATION. DO NOT try to direct it!!!!

B. Question 8: Presents some problems in a different way. No mention is made of the number of blocks which should be used to get the board in balance. In all previous questions the number of blocks to be used was indicated. Initially THIS WILL CONFUSE the learners. DO NOT be too eager to render assistance or explain. Suspend your judgments and "helpfulness."

C. Questions 9, 10, 11, 12, 13, 14 and 15: These questions ask the learners to make "GUESSES." Basically, these questions are trying to determine if the learners can use the collection of experience gained from earlier questions.
NOTE: OBSERVE the learners closely. If a child makes a guess which does not result in the board remaining in balance, watch very closely what he does to correct the situation. If you are unable to observe what a child does, retrace his actions through discussion.

For example, in question 10, if the learner places two blocks on position 2, the board will NOT balance. The important reaction to observe is WHAT THIS CHILD DOES TO CORRECT THE SITUATION.

Does he immediately move these blocks out farther from the fulcrum without any hesitation?

Does he seem confused or uncertain as to what direction to move the blocks to attain balance?

If there is no hesitation in moving the blocks out further, this probably indicates that past experience in earlier questions has been meaningful.

If the child is uncertain, as revealed by his actions, this could mean he has not gained enough experience in the "balancing situation."

D. Question 16: Suggests that the balancing problem be tried when the fulcrum is NOT directly under the midpoint. This may seem difficult for some learners.

E. Question 17: It is important to give each learner an opportunity to develop a "balancing rule" as requested in this question. Very few can develop a QUANTITATIVE RULE but most can express a QUALITATIVE RULE; for example:

E. (continued)......

A few blocks placed far from the fulcrum can balance several blocks placed close to the fulcrum.

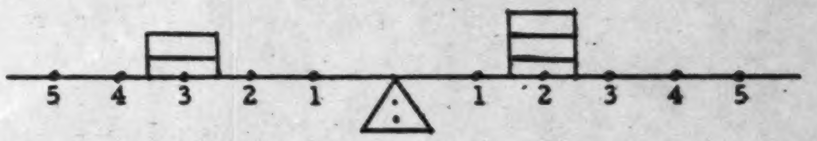
Some learners may notice that:

When the same number of blocks are used on each side of the fulcrum they are always positioned on the same number.

The "balancing rule" is a MULTIPLICATION RULE:

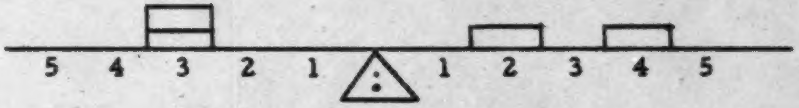
The sum of the multiplication of each block times its respective distance from the fulcrum on one side of the fulcrum must be equal to the same sum on the other side of the fulcrum. For example:

(a)



2 blocks x 3 spaces = 6 3 blocks x 2 spaces = 6

(b)



2 blocks x 3 spaces = 6

1 block x 2 spaces = 2
1 block x 4 spaces = 4
Thus: 2 + 4 = 6

NOTE: Very few primary learners will develop this quantitative rule, but they all can work at developing some qualitative rule which indicates their general understanding of the "balancing situation."

F. High interest can be generated in this activity. After completion of the work with the boards you may be able to obtain one large plank (1" x 12" x 12 feet long) and a block of wood (to serve as a fulcrum--a small round fence post about 12" to 14" long, nailed to a 2" x 4" will serve nicely--and try some of the questions using children as the "blocks."

G. Some children may talk about machines that act like the balancing board. This could

G. (continued).....be an easy way to move into discussions about simple machines--hopefully with the children supplying most of the information, ideaing and "guessing" and the teacher "helping out" with questions of the type suggested in the unit.

H. You may discover ways of using the "marked balancing board" as a NUMBER LINE.

I. "Balancing Blocks" will present many opportunities for all forms of CHILD-CENTERED EXPRESSION. ADD ideas which you create as you "experience" your own learning:

1.

2.

3.

4.

5.

6.

7.

8.

9.

10.

Written Lesson Sheet

BALANCING-BLOCK--QUESTIONS: (NOTE: Record your responses to the questions on the worksheet).

1. BALANCE the board on the fulcrum.
   PLACE 2 blocks over the fulcrum.
   Is the board still in balance?
   Where else can you PLACE 2 blocks and still keep the board in balance?
   RECORD as many answers as you can.

2. BALANCE the board on the fulcrum.
   PLACE 2 blocks on one side of the fulcrum.
   Can you balance the board with 2 more blocks?
   How many different ways can you balance 2 blocks on one side of the fulcrum?

3. BALANCE the board on the fulcrum.
   PLACE 3 blocks on one side of the fulcrum.
   Can you balance the board with 3 more blocks?
   How many different ways can you balance three blocks on one side of the fulcrum with 3 blocks on the other side of the fulcrum?

4. BALANCE the board on the fulcrum.
   PLACE 4 blocks on one side of the fulcrum.
   Can you balance the board with 4 more blocks?
   How many different ways can you balance 4 blocks on one side of the fulcrum with 4 blocks on the other side of the fulcrum?

5. BALANCE the board on the fulcrum.
   PLACE 2 blocks on one side of the fulcrum.
   Can you balance the board with 1 block on the other side of the fulcrum?
   How many different ways can you balance two (2) blocks on one side of the fulcrum with one (1) block on the other side of the fulcrum?

6. BALANCE the board on the fulcrum.
   PLACE 3 blocks on one side of the fulcrum.
   Can you balance the board with 1 block on the other side of the fulcrum?
   How many different ways can you balance three blocks one one side of the fulcrum with 1 block on the other side of the fulcrum?

7. BALANCE the board on the fulcrum.
PLACE 3 blocks on one side of the fulcrum.
Can you balance the board with 2 blocks on the other side of the fulcrum?
How many different ways can you balance 3 blocks on one side of the fulcrum with 2 blocks on the other side of the fulcrum?

8. (a) BALANCE the board on the fulcrum.
PLACE 3 blocks on position 3 on one side of the fulcrum.
What would you do to balance these blocks?
WRITE down some of the answers you get.

(b) BALANCE the board on the fulcrum.
PLACE 4 blocks on position 4 on one side of the fulcrum.
What would you do to balance these blocks?
WRITE down some of the answers you get.

(c) BALANCE the board on the fulcrum.
SPREAD 4 blocks out at different places on one side of the fulcrum.
What would you do to balance these blocks?
WRITE down some of the answers you get.

9. If 1 block was PLACED at position 2 one side of the fulcrum, GUESS where you would PLACE 1 block on the other side of the fulcrum.
WRITE down your guess.
TRY it and see if your guess was correct.

10. If 2 blocks were PLACED at position 3 on one side of the fulcrum, GUESS where you would PLACE 2 blocks on the other side of the fulcrum.
WRITE down your guess.
TRY it and see if your guess was correct.

11. If 3 blocks were PLACED at position 4 on one side of the fulcrum, GUESS where you would PLACE 3 blocks on the other side of the fulcrum.
WRITE down your guess.
TRY it and see if your guess was correct.

12. If 1 block was PLACED at position 4 on one side of the fulcrum, GUESS where you would PLACE 2 blocks on the other side of the fulcrum.
    WRITE down your guess.
    TRY it and see if your guess was correct.

13. If 2 blocks were PLACED at position 3 on one side of the fulcrum, GUESS where you would PLACE 3 blocks on the other side of the fulcrum.
    WRITE down your guess.
    TRY it and see if your guess was correct.

14. GUESS where you must place 2 blocks on one side of the fulcrum to balance 3 blocks OUT TWO SPACES on the other side of the fulcrum.

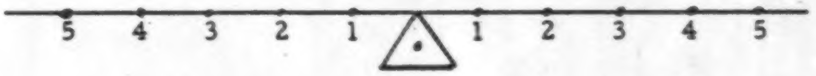
15. GUESS where you must PLACE 4 blocks on one side of the fulcrum to balance 4 blocks OUT FIVE SPACES on the other side of the fulcrum.

16. BALANCE the board on the fulcrum.
    MOVE the board to the RIGHT OR LEFT until the fulcrum is under position 1 on the board.
        Can you balance the board when the fulcrum is under position 1 on the board?
        Can you balance the board when the fulcrum is under position 2 on the board?
    TRY both.

17. You have been balancing the board in many different ways; sometimes you used the same number of blocks on each side of the fulcrum and sometimes you did not.
        Can you think of a GENERAL RULE for balancing the board with the blocks?
            FIRST TRY to make a general rule for the times you used the same number of blocks on each side of the fulcrum.
                Will your rule work for the case when there are a different number of blocks on each side of the fulcrum?

BALANCING-BLOCK MATH

WORKSHEET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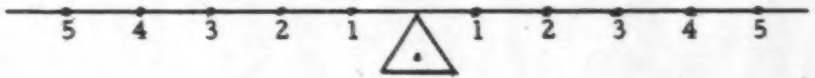
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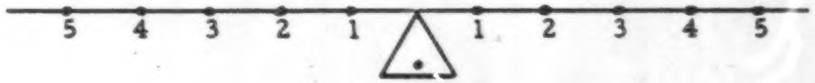
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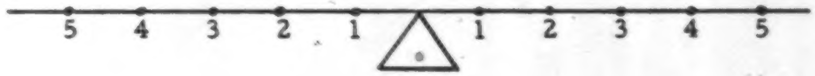
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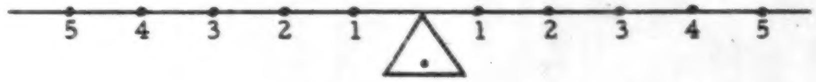
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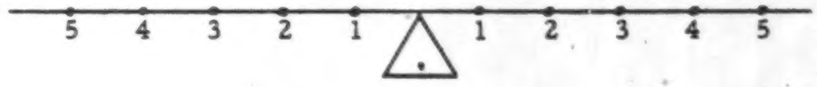
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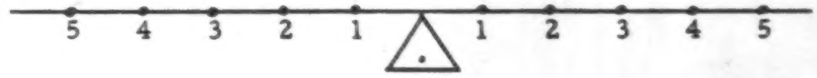
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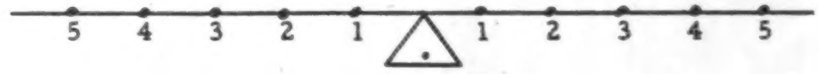
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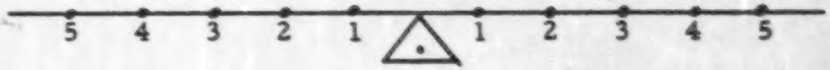
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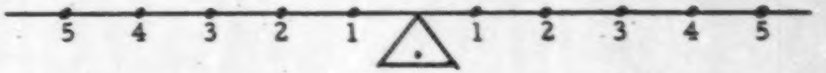
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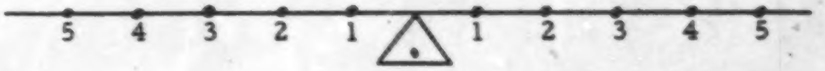
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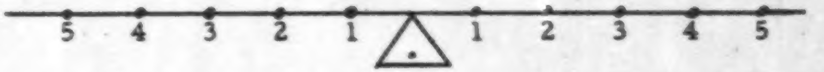
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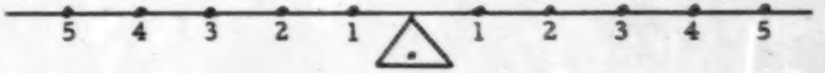
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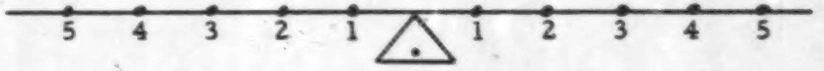
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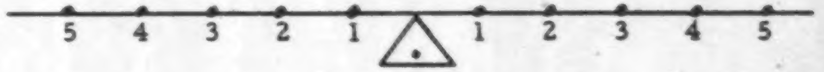
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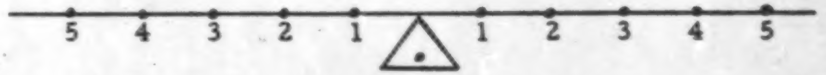
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